Prelim 2021 - BUSI899-040 - Macro and Asset Pricing

In this question we will analyze the effect of physical investment and R&D investment on expected stock returns. Consider a model in which a firm owns a stock of physical capital, K_t, such as machines and equipment, and a a stock of intangible capital, N_t, such as patents and blueprints. A fraction $\lambda \in (0,1)$ of the intangible capital is used for the production of output, Y_t, given by:

$$Y_t = X_t K_t^{\alpha} \left(\lambda N_t\right)^{1-\alpha},$$

where X_t is productivity. Note that the production technology features constant returns to scale. The stock of intangible capital evolves as:

$$N_{t+1} = (1-\delta_n)N_t + S_t, \tag{1}$$

where δ_n is the depreciation rate of intangible capital, and S_t is R&D investment. Assume that a proportion of $1-\lambda$ of the intangible capital is used to produce new physical capital. That is, the stock of physical capital evolves as:

$$K_{t+1} = (1-\delta_k)K_t + \Phi(I_t, (1-\lambda)N_{t+1}), \tag{2}$$

where I_t is physical investment, and $\Phi(\cdot,\cdot)$ denotes the production of new physical capital units. Assume that $\Phi_{1,t} \equiv \Phi_1(I_t, (1-\lambda)N_{t+1})) > 0$, and $\Phi_{2,t} \equiv \Phi_2(I_t, (1-\lambda)N_{t+1})) > 0$. Put differently, the production of new physical capital increases in the level of physical investment (as standard in production models), and also in the stock of intangible capital (i.e., more technological progress, or more blueprints, make physical investment more productive). The function Φ is constant returns to scale in its inputs, and therefore Φ can be re-written as:

$$\Phi_t \equiv \Phi(I_t, (1-\lambda)N_{t+1}) = I_t\Phi_{1,t} + (1-\lambda)N_{t+1}\Phi_{2,t}. \tag{3}$$

The dividend D_t is given by output net of physical and intangible investment, $Y_t - I_t - S_t$. The firm's problem can be expressed recursively as:

$$V_t = V(K_t, N_t, X_t) = \max_{K_{t+1}, I_t, N_{t+1}, S_t} Y_t - I_t - S_t + \mathbb{E}_t[M_{t,t+1}V_{t+1}]$$
$$\text{s.t. (1), (2)}$$

Let q_t^N be the Lagrangian multiplier on (1), and q_t^k be the Lagrangian multiplier on (2). Define $\tilde{q}_t^N = 1 - q_t^K(1-\lambda)\Phi_{2,t}$. If you cannot solve a particular item below move to the next one, and use the result(s) from the previous items as given.

(a) The optimality conditions can be written as:

$$\mathbb{E}_t[M_{t,t+1}R_{t+1}^K] = 1, \quad \mathbb{E}_t[M_{t,t+1}R_{t+1}^N] = 1,$$

where R_{t+1}^K is the return on physical investment and R_{t+1}^N is the return on intangible capital.

Use the first order conditions with respect to I_t, S_t, K_{t+1}, and N_{t+1}, to show that:

$$R^K_{t+1} = \frac{\alpha \frac{Y_{t+1}}{K_{t+1}} + (1-\delta_K) q^K_{t+1}}{q^K_t}, \quad R^N_{t+1} = \frac{(1-\alpha)\frac{Y_{t+1}}{N_{t+1}} + (1-\delta_N)}{\tilde{q}^N_t}.$$

(b) Use the identity $Y_t = \alpha \frac{Y_t}{K_t} K_t + (1-\alpha)\frac{Y_t}{N_t} N_t$, equation (3), and the first-order condition with respect to I_t, to show that:

$$\begin{aligned} V_t = &\left[\alpha \frac{Y_t}{K_t} + q^K_t (1-\delta_K)\right] K_t + \left[(1-\alpha)\frac{Y_t}{N_t} + (1-\delta_N)\right] N_t \\ &- q^K_t K_{t+1} - \tilde{q}^N_t N_{t+1} + \mathbb{E}_t[M_{t,t+1} V_{t+1}] \end{aligned}$$

(hint: similar steps to the initial derivation of q-theory, as we did in class)

(c) Assume the following transversality conditions hold:

$$\lim_{\tau\to\infty} \mathbb{E}_t M_{t,t+\tau} q^K_{t+\tau} K_{t+\tau+1} = 0, \quad \lim_{\tau\to\infty} \mathbb{E}_t M_{t,t+\tau} \tilde{q}^N_{t+\tau} N_{t+\tau+1} = 0.$$

Use the result in (b), the first order conditions, and recursive substitution to show that:

$$V_t = \left[\alpha \frac{Y_t}{K_t} + q^K_t (1-\delta_K)\right] K_t + \left[(1-\alpha)\frac{Y_t}{N_t} + (1-\delta_N)\right] N_t$$

(d) Using the result in (c) and equation (3) show that:

$$V_t = D_t + q^K_t K_{t+1} + \tilde{q}^N_t N_{t+1}$$

(e) Define the stock return as $r^S_{t+1} = \frac{V_{t+1}}{V_t - D_t}$. Using the result in (d), show that $\mathbb{E}_t[r^S_{t+1}]$ is a weighted average between the expected return on physical capital and the expected return on intangible capital:

$$\mathbb{E}_t[r^S_{t+1}] = w^K_t \mathbb{E}_t[R^K_{t+1}] + w^N_t \mathbb{E}_t[R^N_{t+1}],$$

and find w^K_t and w^N_t (*Note that K_{t+1} and N_{t+1} are time-t measurable*).

(f) (i) Does R&D investment increase or decrease the expected return on physical capital?

(ii) Does R&D investment increase or decrease the expected return on intangible capital?

(iii) If the price of physical capital is high (i.e., high q^K_t), does the firm's expected return increase or decrease in R&D investment?