

Internet Appendix for “Trendy Business Cycles and Asset Prices”

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IA.1 Equity premium based on a consumption claim

For comparison, we derive the production-based model-implied expected excess return for an asset whose dividend growth follows the path of consumption, i.e., ΔC_t . This exercise is similar to estimating the equity premium implied by endowment economies, in which the dividend growth dynamics are assumed to be equal to (levered) consumption growth.

Unconditionally, the model-implied expected excess return of a consumption claim is 6.3% under full-information, but 8.9% under imperfect-information. In this case, the asset’s risk exposure to short- and long- run consumption risk is exogenous, and positive by construction and so the model-implied risk premium is positive even under full-information. This highlights the challenge that our benchmark model faces in generating a sizable equity premium: endogenous investment changes the sign and the magnitude of risk exposure(s). However, learning still amplifies the risk premium for such an asset by creating more endogenous long-run consumption risk (or greater persistence in expected consumption growth). As a result, the risk premium under imperfect-information is 30% larger than under full-information.

Similar to our analysis of the dividend claim, we find that trend shocks drive the risk premium under full-information, while transitory shocks play this role under learning. Specifically, we estimate that over 99% of the risk premium is due to permanent shocks, ε_x , whereas under imperfect-information,

this flips: now, over 99% of the premium is explained by transitory shocks ε_z .

In production frameworks, dividends are typically countercyclical because of the smoothing effect of investment. In contrast, as in endowment economies, this exercise demonstrates that the model-implied risk premium can be considerably larger when the correlation between dividend and consumption is positive. It also suggests ways that our model could generate a higher equity premium, for instance, by featuring asymmetric adjustment costs, which makes it harder to smooth profits in bad states (making dividends less countercyclical; see, e.g., Zhang (2005)). We abstract from more elaborate adjustment costs for parsimony, since our main interest is not the absolute value of the equity premium, but rather its relative magnitude compared to full-information.

IA.2 Robustness to model parameters

The asset pricing implications are largely robust to changes in key parameters that govern the technology process and preferences. Table IA.4.1 in the Internet Appendix details the key model-implied moments, obtained from the full-information and imperfect-information models. The calibration is identical to that outlined in Table 1 apart from σ_z (Panel A), ρ_z (Panel B), or ψ (Panel C). Figures IA.4.1, IA.4.2, and IA.4.3 in the Internet Appendix show impulse responses of transitory and permanent shocks to macroeconomic variables under these alternative choices for σ_z , ρ_z and ψ , respectively.

Before describing the impact of these changes in detail, we note that in all cases, the the model's main predictions hold: under learning, business cycle shocks (i) explain almost the entire magnitude of the equity premium, and (ii) correlate positively with long-run consumption risk shocks. By contrast, under full-information, these transitory shocks correlate negatively with endogenous long-run risk.

In Panel A of Table IA.4.1 we consider the cases in which the business-cycle shock is more or less volatile than the benchmark. In both cases, the equity premium is negative under full-information but positive under imperfect-information. In particular, when σ_z is about twice as large, the implied equity premium is considerably more sizable, at about 6.5%. While a more sizable business shock implies that a smaller fraction of ε_z is attributed to the trendy business-cycle component (i.e., g_x/g_z falls), the absolute volatility of these shocks become larger. Consequently, consumption growth's volatility increases, although it remains realistic, at just above 2% per annum. Figures IA.4.1 shows that when σ_z is increased or decreased, most impulse-responses are qualitatively similar to the benchmark case. The

only notable difference is that when σ_z is sufficiently large, a business-cycle shock can drop expected consumption growth under imperfect-information (see Figure IA.4.1 (e)), whereas in the benchmark case, this shock raises this quantity. Nonetheless, the drop in expected consumption growth is muted under imperfect-information in comparison to full-information.

Panel B of Table IA.4.1 shows that under learning, the flip in the equity premium's sign still hold whether the business-cycle component is not persistent at all ($\rho_z = 0$) or as persistent as the trend component ($\rho_z = \rho_x$). The magnitude of the equity premium under imperfect-information is somewhat larger when ρ_z falls. A drop in ρ_z makes it harder for the agent to distinguish between the two types of shocks (since their unconditional volatilities are now more similar), and therefore, a larger portion of each business-cycle shock is absorbed in the trendy component. This happens despite of the sharp difference in the persistence between the z and x processes. Because persistence can only be observed as time passes, but not in real time, the persistence of z matters to the results only to the extent that it affects the magnitude of the volatility of the z process in comparison to that of the x process.

Lastly, in Panel C of Table IA.4.1 we consider a case in which the IES is very high ($\psi = 2$) and a case in which the IES is below 1 ($\psi = 0.9$). In both cases, the equity premium under imperfect-information is positive (because business-cycle shocks raise investment in both cases, which is consistent with positive risk exposure), and larger in absolute value compared to full-information, in which the equity premium is negative. When the IES is higher, the wealth effect gets weaker. As a result, the agent does not drop investment as much in response to a trend shock. This suggests that under full-information and $\psi = 2$, the exposure of firms to trend shocks is less negative, and the equity premium is closer to zero. The lower the IES, the smaller the effect of the continuation utility on the marginal utility (this depends on $\gamma - \frac{1}{\psi}$), and consequently, risk premia are smaller.

IA.3 Parametrization using Kalman Filtering

In our benchmark estimation, we utilize the simulated method of moments to target key moments in the macroeconomic data. An alternative approach is to calibrate the parameters driving the technology process to realized productivity growth. For example, Basu, Fernald and Kimball (2006) provide an estimate of quarterly TFP growth using value added, hours worked, and a the stock of different facets of capital. For our benchmark estimation of the model we refrain from using the TFP data directly

because TFP growth in the data and productivity growth, ΔA , in the model can differ along two dimensions. First, Basu et al (2006) use ten types of capital, and some of those types correspond to organizational (intangible) capital (e.g., artistic capital, R&D capital) which do not have an equivalent in our model. Second, the TFP data is only partially adjusted for utilization fluctuations and is not adjusted for time-varying markups. Consequently, measured TFP growth may not be driven exclusively by technological changes, as in our setup.

Despite these potential shortcomings, we show that the model parameters and the associated model-implied moments are materially similar utilizing this alternative approach. As a first step, we find the set of parameters $\{\sigma_z, \sigma_x, \rho_z, \rho_x\}$ that maximize the log-likelihood of the filtered TFP growth time-series using the state-space implied by Equation (10). The results are reported in Panel A of Table IA.4.2. There are two important similarities between the estimates and the benchmark parametrization in Table 1: (i) business cycle shocks are more sizable, i.e., σ_z/σ_x is about 24 versus 21 in the benchmark (the two estimates σ_z and σ_x are statistically distinct), and (ii) both the business cycle state and the trend state are persistent.

A summary of the key moments under learning and full-information is shown in Panel B of Table IA.4.2 using the maximum-likelihood technology parameters, while holding fixed the other parameters in Table 1. As in the baseline results, endogenous long-run consumption risk shocks correlate positively with business cycle shocks under imperfect-information, and most of the equity premium is attributed to business-cycle fluctuations. The volatility of consumption growth is only about 1/3 of its empirical counterpart, however and consequently, the equity premium is lower than predicted in our benchmark despite still being four times larger than the full-information estimate. To target the volatility of consumption growth while keeping the relative magnitude of trend/business-cycle shocks constant, we re-scale both σ_x and σ_z by the same constant, $b = 2.3$. While the other moments are essentially unchanged, this leads to an equity premium near 2% under learning.

IA.4 Supplemental Tables and Figures

Table IA.4.1: **Key Model-Implied Moments Under Alternative Calibrations**

	Panel A: Alternative σ_z			
	high σ_z		low σ_z	
	Imperfect Information	Perfect Information	Imperfect Information	Perfect Information
$Corr(\Delta c_t - E_{t-1}\Delta c_t, \varepsilon_{z,t})$	0.68	-0.85	0.89	-0.62
$\beta_z \lambda_z / \text{Equity Premium}$	99.73%	-59.71%	98.75%	-8.41%
$E[R_m^e]$	6.59	-0.11	0.83	-0.66
$SR(R_m^e)$	0.88	-0.01	0.20	-0.15
$\sigma[r_f]$	0.63	0.90	0.67	0.67
$\sigma[\Delta C]$	2.56	2.34	1.19	1.13

	Panel B: Alternative ρ_z			
	$\rho_z = \rho_x$		$\rho_z = 0$	
	Imperfect Information	Perfect Information	Imperfect Information	Perfect Information
$Corr(\Delta c_t - E_{t-1}\Delta c_t, \varepsilon_{z,t})$	0.97	-0.51	0.68	-0.86
$\beta_z \lambda_z / \text{Equity Premium}$	99.67%	-86.26%	99.69%	-15.01%
$E[R_m^e]$	2.30	-0.17	3.29	-0.58
$SR(R_m^e)$	0.47	-0.04	0.65	-0.12
$\sigma[r_f]$	0.67	0.69	0.96	1.02
$\sigma[\Delta C]$	1.65	1.58	1.16	1.10

	Panel C: Alternative ψ			
	$\psi = 2$		$\psi = 0.9$	
	Imperfect Information	Perfect Information	Imperfect Information	Perfect Information
$Corr(\Delta c_t - E_{t-1}\Delta c_t, \varepsilon_{z,t})$	0.39	-0.87	0.89	-0.74
$\beta_z \lambda_z / \text{Equity Premium}$	99.68%	-12.09%	99.69%	-10.15%
$E[R_m^e]$	2.67	-0.53	1.65	-1.02
$SR(R_m^e)$	0.49	-0.10	0.38	-0.20
$\sigma[r_f]$	0.41	0.55	1.08	1.13
$\sigma[\Delta C]$	1.32	1.49	2.05	1.46

The table shows key model-implied moments, both under full-information and imperfect-information, when the calibration is identical to that outlined in Table 1, apart from σ_z (Panel A), ρ_z (Panel B), or ψ (Panel C).

Table IA.4.2: **Filtering TFP Data: Parameters and Model-Implied Moments**

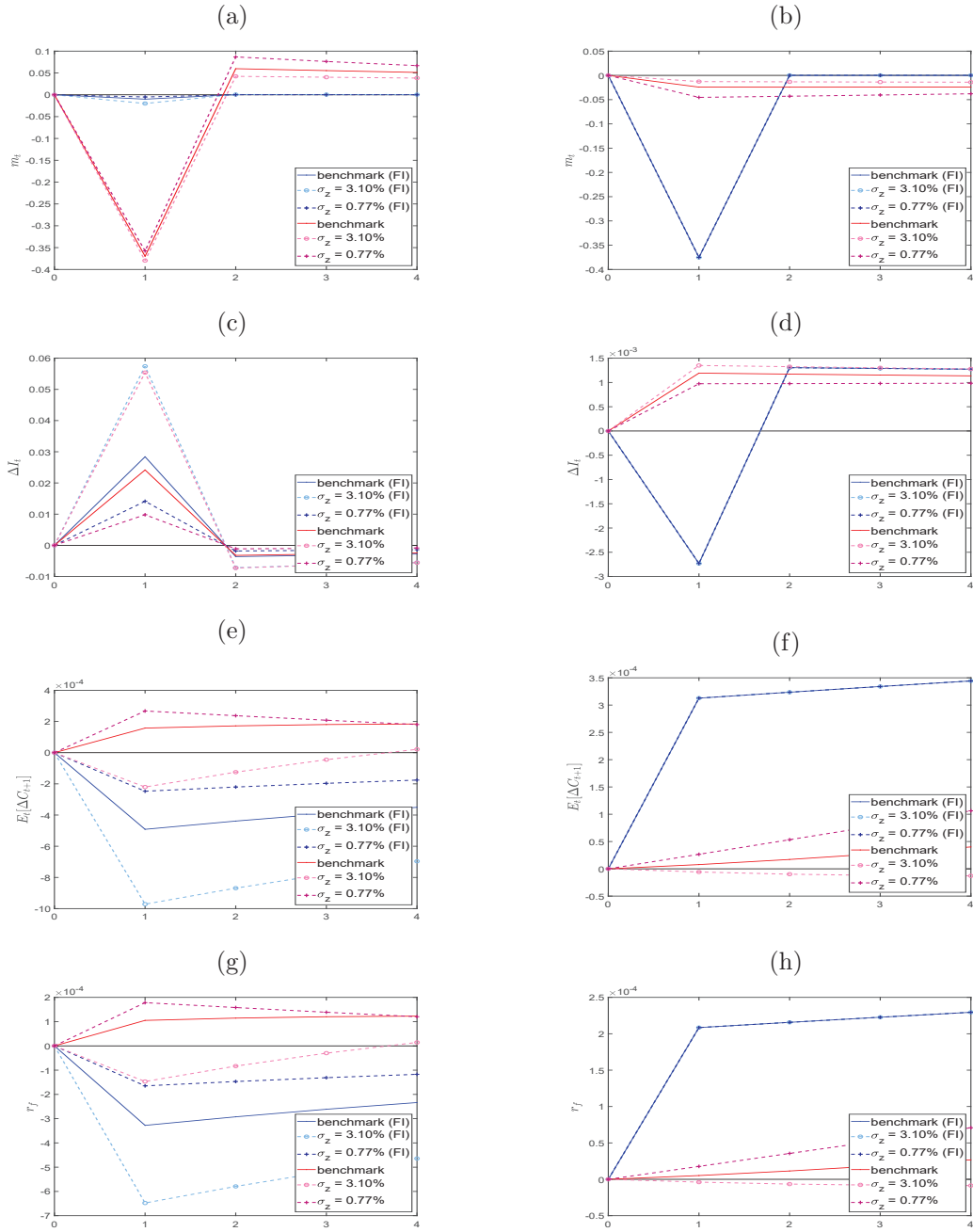
Panel A: Estimated parameters				
	σ_z	σ_x		
	0.00884	$0.35640 \cdot 10^{-3}$		
	[0.00806,0.00962]	$[0.000004,0.77078] \cdot 10^{-3}$		
	ρ_z	ρ_x		
	0.933	0.981		
	[0.757,0.990]	[0.932,0.993]		
Panel B: Moments				
	$b = 1$		$b = 2.3$	
	Imperfect Information	Perfect Information	Imperfect Information	Perfect Information
$Corr(\Delta c_t - E_{t-1}\Delta c_t, \varepsilon_{z,t})$	0.97	-0.07	0.96	-0.16
$E[R_m^e]$	0.44	0.13	1.89	0.31
$SR(R_m^e)$	0.10	0.03	0.37	0.06
$\sigma[r_f]$	0.20	0.23	0.44	0.51
$\sigma[\Delta C]$	0.69	0.62	1.59	1.43

Panel A shows the estimated technology parameters using Kalman Filter MLE and Basu, Fernald and Kimball (2006) quarterly TFP growth data. Panel B shows key model-implied moments, both under full-information and imperfect-information, when the calibration is identical to that outlined in Table 1, apart from the parameters estimated in Panel A. In the right-most columns, σ_z and σ_x are scaled by the factor b so that the volatility of consumption in the model matches its empirical counterpart.

Figure IA.4.1: Sensitivity Analysis: BC Volatility

Impulse-Responses: Transitory Shock (ε_z)

Impulse-Responses: Permanent Shock (ε_x)

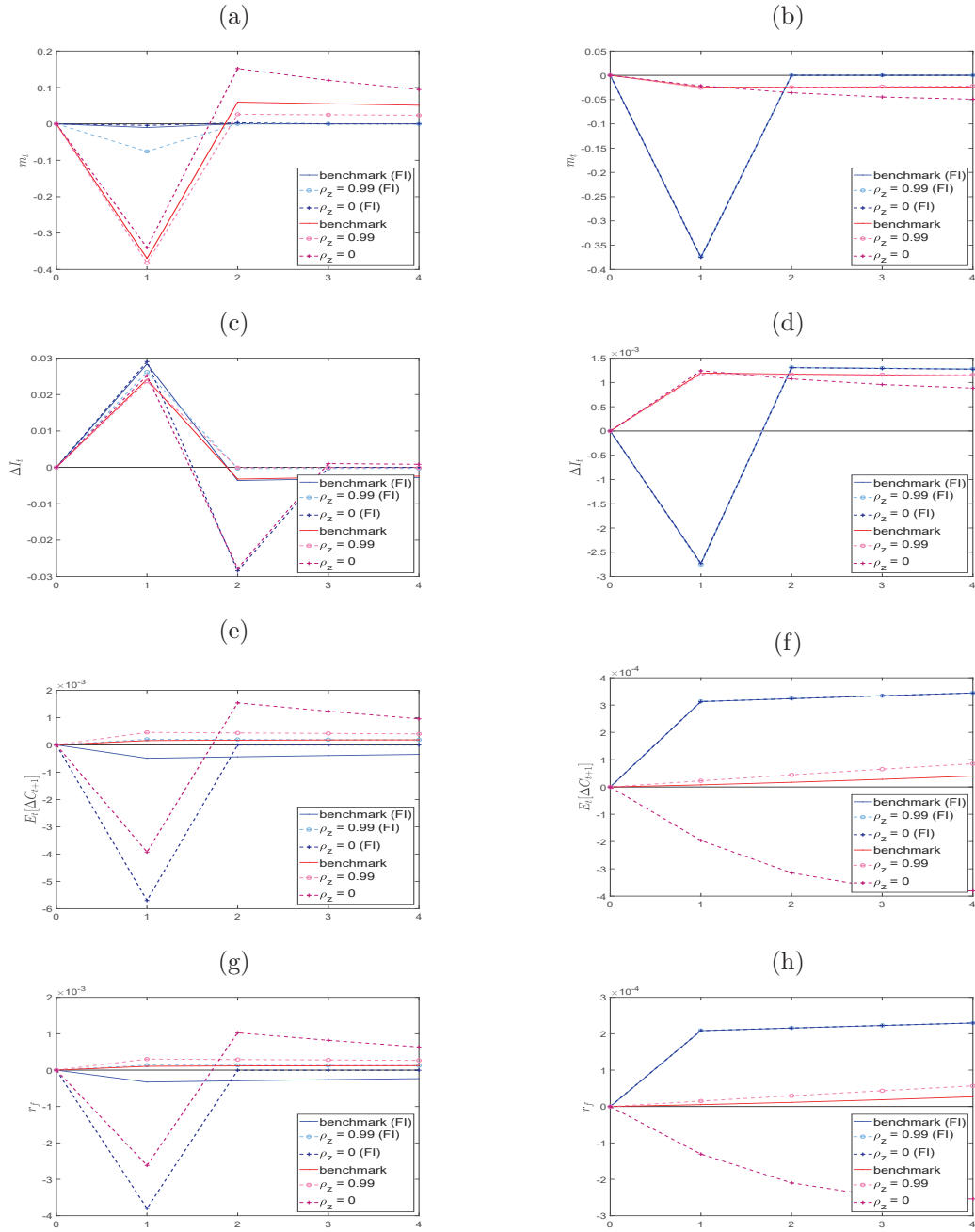


The figure shows impulse responses of transitory and permanent shocks to macroeconomic variables under alternative choices for σ_z .

Figure IA.4.2: Sensitivity Analysis: BC Persistence

Impulse-Responses: Transitory Shock (ε_z)

Impulse-Responses: Permanent Shock (ε_x)

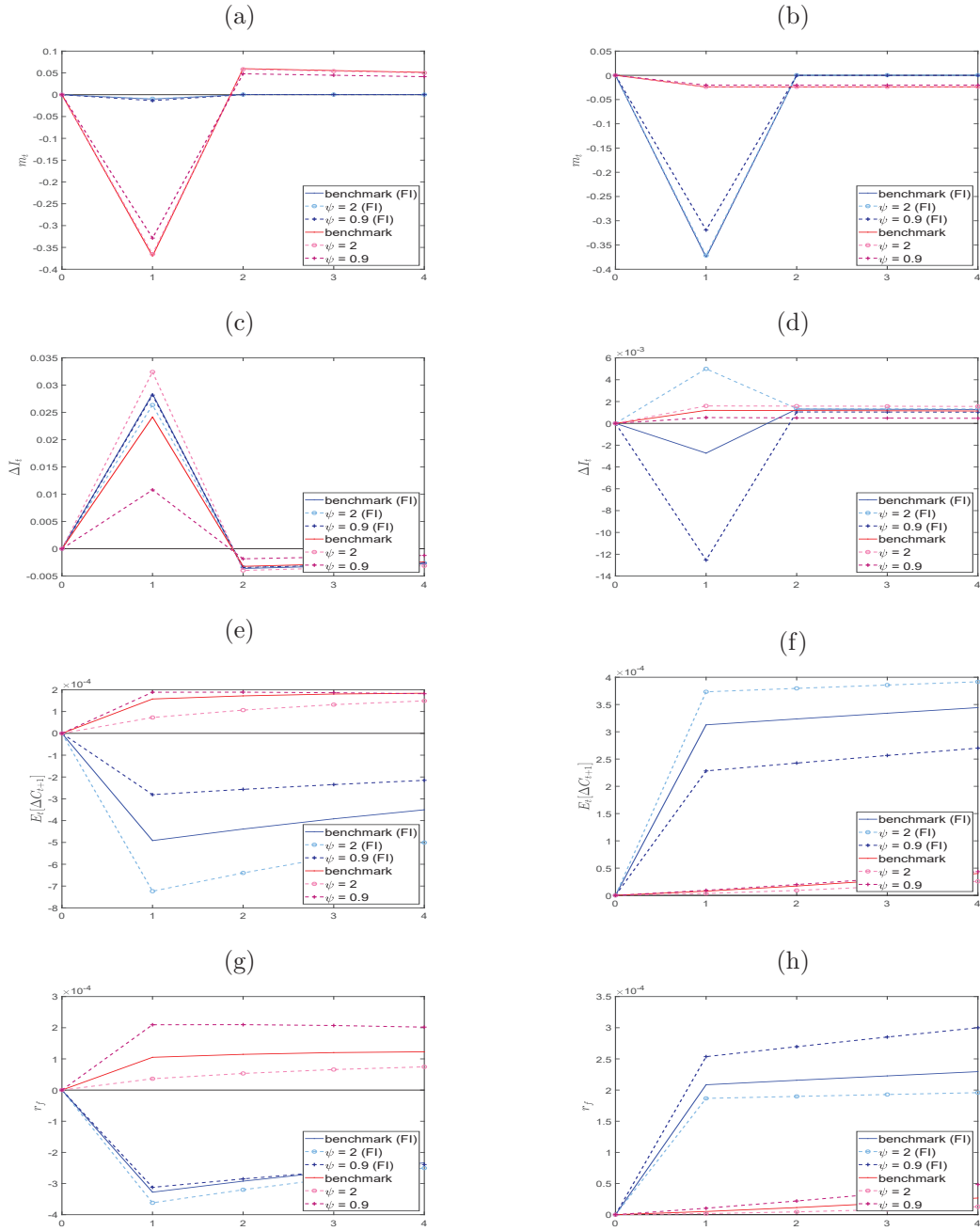


The figure shows impulse responses of transitory and permanent shocks to macroeconomic variables under alternative choices for ρ_z .

Figure IA.4.3: Sensitivity Analysis: IES

Impulse-Responses: Transitory Shock (ε_z)

Impulse-Responses: Permanent Shock (ε_x)

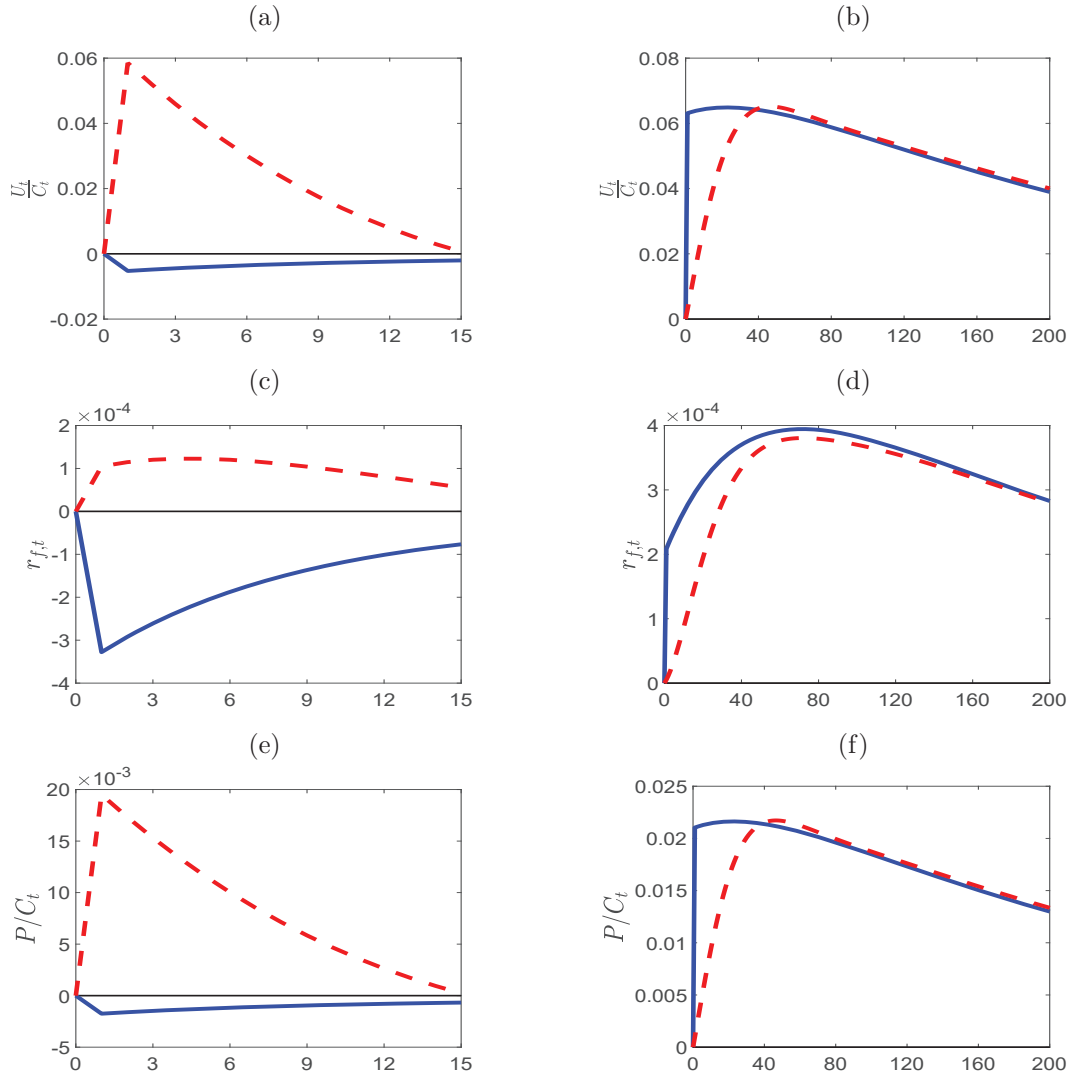


The figure shows impulse responses of transitory and permanent shocks to macroeconomic variables under alternative choices for the IES value.

Figure IA.4.4: Price Dynamics Under Learning - Long Horizon IR

Impulse-Responses: Transitory Shock (ε_z)

Impulse-Responses: Permanent Shock (ε_x)



The figure shows impulse responses of transitory and permanent shocks to financial (prices) variables under risk aversion, for longer predictive horizons. The solid blue line shows impulse-responses under full-information. The dashed red line shows impulse-responses under learning.