

“Uncertainty, Risk, and Capital Growth” *

Internet Appendix

IA.1 Data and Measurements: Detailed Discussion

IA.1.1 Benchmark Measures

We first describe the data and methodology to construct our benchmark measures (labeled ‘Benchmark’ henceforth) of capital accumulation, productivity, and macroeconomic uncertainty. For robustness, we consider alternative approaches to measure our key economic variables, presented in the subsequent section.

Capital. The BEA Fixed Assets Accounts provide quantity indices for the net stock, economic depreciation, and investment in fixed assets. Capital stock and investment measures provided by the BEA are routinely used in macroeconomic research to define variables of interest (e.g., investment-to-capital or output-to-capital ratios) or in productivity computations (Fernald, 2014). While it is less common to consider depreciation rates separately, it is important to highlight that capital stock, investment, and depreciation are directly related through the capital accumulation equation, which is satisfied under the BEA measurement. Thus, the three measures are on par in terms of their economic and empirical applicability. We use the capital accumulation restriction to tie in the components of capital accumulation in our empirical analysis, as described in Section 1.2.2.

All fixed asset indices are chained to the year 2012. We convert the indices to real dollars by multiplying them by their respective dollar amount as of 2012. Depreciation and investment rates are constructed as the dollar depreciation and investment in the current year, respectively, divided by the stock of capital at the end of the previous year. In the benchmark, these measures correspond to private non-residential capital. Based on availability of fixed assets data, all variables are sampled at an annual frequency from 1948 to 2022.

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Productivity. The main driving forces in our theoretical framework in Section 2 are related to the first and second moments of aggregate productivity. To ensure a tight connection between the model and the data, in our benchmark analysis, we use the utilization-adjusted measure of total factor productivity (TFP) growth, constructed by [Basu, Fernald, and Kimball \(2006\)](#) and [Fernald \(2014\)](#) and available at the San Francisco Fed, as a proxy for aggregate productivity (that is, its first moment).

Uncertainty. Consistent with our economic model, we measure second-moment shocks through the supply-side uncertainty, and more specifically, the time-varying conditional volatility of productivity shocks.¹

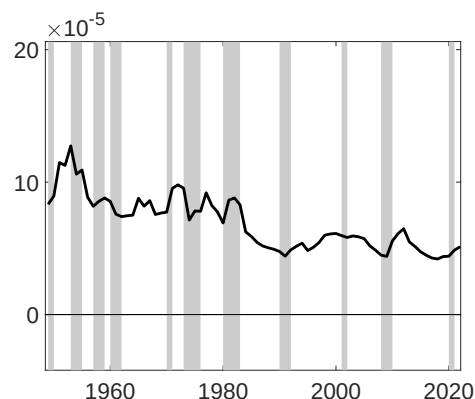
To construct our benchmark measure of macroeconomic variance, we estimate a parametric AR(1)-GARCH model. The AR(1) component strips away persistent variations in expected productivity and allows us to identify innovations in the TFP. We filter out the time-varying volatility of these innovations via a conditional heteroskedasticity model. In the benchmark, we employ the Glosten-Jagannathan-Runkle ($GJR(p, q)$) model. It nests a standard *GARCH* framework, but accounts for an asymmetry in the volatility response to positive and negative TFP shocks, and thus helps capture the countercyclical movements of macroeconomic volatility over the business cycle. For accurate measurements of volatility, we use the highest frequency available for the utilization-adjusted TFP data, which is quarterly. We set $p = 4$ and $q = 1$ to account for any seasonal effects in quarterly data and take the end of the fourth quarter volatility estimate to form annual observations.²

Figure [IA.1.1](#) shows our benchmark measure of the macroeconomic uncertainty; the time series is smoothed over the last 3 years to reduce high-frequency oscillations. The volatility measure is countercyclical, typically showing a persistent increase during the NBER recessions.

¹From a theoretical perspective, fluctuations in supply-side uncertainty relate to the stochastic volatility of permanent productivity shocks, and, consequently, induce a sizable and persistent impact on production allocations, consumption decisions, and the marginal utility of economic agents. In contrast, while sizable, firm-level uncertainty does not constitute a systematic source of risk in complete markets. Similarly, facets of aggregate demand-side uncertainty typically govern the volatility of *mean-reverting* shocks to preferences, taxation, or sentiments. Such transitory shocks play a limited role for the aggregate risk premium as well (see, e.g., [Alvarez and Jermann, 2005](#)).

²The results are robust to taking the average volatility over the last four quarters, although this alternative methodology potentially contaminates the latest conditional volatility with previous quarters.

Figure IA.1.1: Macroeconomic uncertainty



The figure shows the time series of the benchmark macroeconomic uncertainty, estimated from the GJR model fitted to the TFP innovations. The shaded areas represent NBER recessions.

Consistent with [Stock and Watson \(2002\)](#), fluctuations in macroeconomic uncertainty are more pronounced in the pre-Volcker period.³

IA.1.2 Alternative Measures

We consider a number of alternative measures to corroborate our main empirical evidence; Table [IA.2.2](#) in the Online Appendix shows the correlation between the different measures of capital, productivity and uncertainty.

Capital Alternatives. Our benchmark is based on private non-residential capital. For robustness, we consider alternative measures of capital stock, investment, and depreciation related to either private nonresidential equipment and structures (labeled ‘Capital: Equip & Struct’) or to total private capital (labeled ‘Capital: Total Priv’). Compared to the benchmark, the former narrows down the scope of capital to tangibles by stripping away stock and investment to intellectual property. The latter broadens the definition of capital by accounting for residential capital. We also consider replacing capital and investment from the BEA fixed assets account with their accounting equivalents from Compustat (labeled ‘Capital: Compustat’). We proxy the aggregate stock of capital by the total amount of Property, Plant and Equipment, *PPENT*, and construct the investment rate as the aggregate capital expenditures, *CAPX*, divided by the lagged value of *PPENT*. We infer the economic impact of depreciation from the observed

³We examine the subsample evidence in our analysis and show that our main empirical results remain similar and even strengthen in the second half of the sample; see ‘Sample: 1965-’ (Column (10)) in Table 2.

capital and investment rates, as the accounting depreciation item is missing for a non-negligible amount of firms in Compustat.

Productivity Alternatives. Following [Basu, Fernald, and Kimball \(2006\)](#), the benchmark utilization-adjusted TFP growth measure is constructed by the San Francisco Fed as $\Delta Y_t - \alpha_t \Delta K_t - (1 - \alpha_t) \Delta L_t - \Delta u_t$, where ΔY is the log growth in business output, ΔL is the log growth of labor inputs, α is capital's share of output, u is the utilization rate, and ΔK is the growth of capital stock. The aggregate stock of capital corresponds to the weighted average of different types of capital, some of which are tangible and some intangible. Since our economic framework formally features tangible capital, we construct two alternative Solow residuals that capture Hicks-Neutral productivity. Specifically, we employ the same methodology as the San Francisco Fed but replace ΔK with the growth in tangible capital. In the first variation, we exclude from ΔK forms of capital that relate to organizational capital (e.g., R&D and artistic stocks, labeled 'Productivity: Solow(Ex R&D)'). In the second variation, we narrow the capital growth definition further and use only the stock of equipment and structures (labeled 'Productivity: Solow(Equip & Struct)'). Lastly, we consider replacing the benchmark TFP measure with labor productivity published by the Bureau of Labor Statistics (labeled 'Productivity: Labor Prod'). Due to a balanced growth assumption, our model features labor-augmenting technology whose impact is akin to labor productivity. Labor productivity measurements are presumably less noisy as they are based on easily observed variables, namely, value-added and hours. Overall, while these alternatives arguably provide a tighter connection to the model, we opt to use them to show the robustness of our results and rely on traditional off-the-shelf measures for the benchmark analysis.

Uncertainty Alternatives. We consider several alternatives to our benchmark GJR model. First, we implement a stochastic volatility (SV) model which features a separate AR(1) process for log volatility with potentially heavy-tailed shocks (scale mixtures of Gaussian distributions). The SV model allows for separate innovations into volatility and nests our volatility specification in the economic model. We use Bayesian MCMC methods to estimate the model via the methodology

of Chan and Hsiao (2014).⁴ We also replace the GJR model with an exponential GARCH (labeled ‘Uncertainty: E-GARCH’) volatility. The E-GARCH specification posits more intricate volatility dynamics by accounting for an asymmetric impact of negative shocks and the logarithmic transformation. To account for seasonality, we maintain the choice of using four lags when filtering the volatility process.

IA.2 Additional Empirical Results

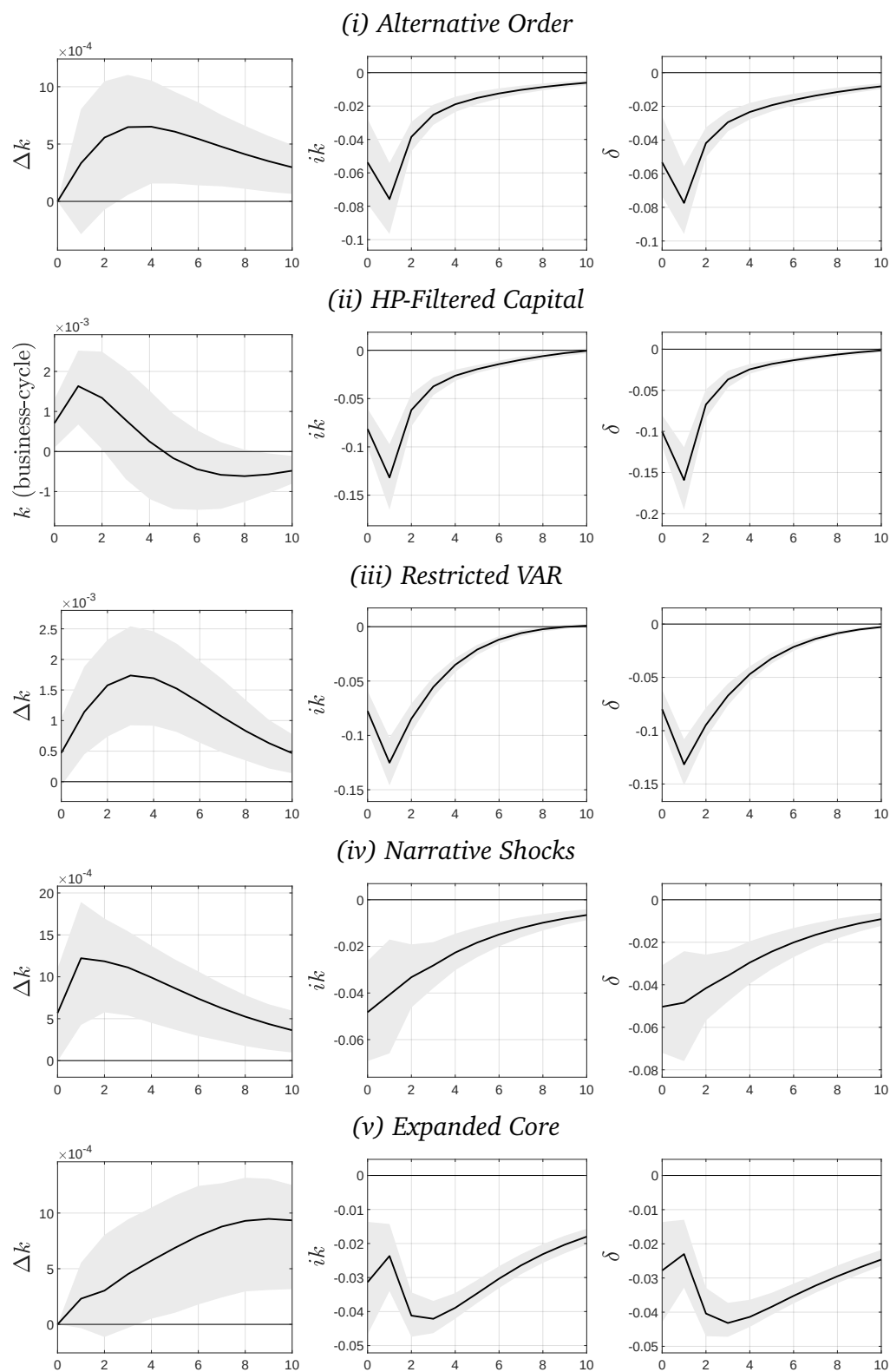
IA.2.1 Structural VAR: Alternative Methodologies

We consider four main methodological variations to the VAR estimation. First, we change the order of the variables in the vector Γ_t to be the growth rate in capital, lagged depreciation, productivity, and lastly, macro uncertainty (labeled ‘Alternative Order’). While under this order, structural uncertainty shocks cannot induce a contemporaneous impact on capital growth, they are effectively orthogonalized from TFP growth and the other core variables. Second, we restrict the estimation of the matrix Φ such that the evolution of TFP growth and its uncertainty only depend on their own lags and are not impacted by the other core variables (labeled ‘Restricted VAR’). While less general, the implied law of motion from the restricted variation parallels to Equations (20) and (21) of the model. Third, we replace the core variable capital growth with the hp-filtered level of k_t (labeled ‘HP-Filtered Capital’).

Next, we maintain the baseline methodology but, in line with Bloom (2009), replace the time series of macro volatility with an indicator taking the value of 1 in years when the volatility rose significantly above the previous year. We use the 80% percentile as a cutoff, given that we work with annual data. We identify this way 11 volatility shocks, and most of them coincide with major turmoil events, such as in the year 1951 (the height of the Korean War), 1953 (Recession caused by a decline in spending after the end of the Korean War), 1965 (Indo-Pakistani War), 1967 (Vietnam war), 1971 (Cambodian Campaign; Fiscal tightening; Off the gold standard), 1975 (Oil crisis peak), 1977, 1981 (Volcker, tight monetary policy), 1992 (mild recession by oil price shock post Gulf War), 1996 (IT revolution onset), and 2010 (end of the Great

⁴See full details and implementation code under <http://people.anu.edu.au/joshua.chan>.

Figure IA.2.2: Uncertainty Shock Impulse Responses: Alternative Methods



The figure shows one-standard-deviation uncertainty shock impulse responses of log capital growth Δk , investment rate ik , and depreciation rate δ , based on alternative methodologies.

Recession). As discussed by [Bloom \(2009\)](#), this indicator arguably identifies exogenous, mainly global, volatility events rather than ongoing fluctuations. This approach combines a Cholesky identification with a narrative identification of shocks (thus, labeled ‘Narrative Shocks’).

Lastly, we consider expanding the core variables in the vector Γ_t to include a host of other controls consistent with [Bloom \(2009\)](#) (labeled ‘Expanded Core’). In this case, Γ_t includes TFP growth, excess market return, Federal Funds Rate, log(CPI) hp-filtered, log(employment) hp-filtered, hours, and macro uncertainty. Uncertainty is ordered last, such that uncertainty shocks are orthogonal to all other controls. In this case, the analysis starts in 1955 due to data availability on the Federal Funds Rate.

Figure [IA.2.2](#) demonstrates that the empirical findings are robust to an alternative order of variables in the VAR, using hp-filtered capital, the Restricted VAR, the Narrative VAR, or the Expanded Core configurations. Under the Narrative Shock VAR, and averaged across all horizons, the impulse response to depreciation is 29% larger, in absolute value, than the impulse response to investment rate. Within the restricted VAR, the cumulative effect of uncertainty on ten-year-ahead capital growth reaches 1.25%.

IA.2.2 Structural Uncertainty Shock Identification

The identification of uncertainty shocks in the benchmark setting relies on a recursive scheme (i.e., Cholesky decompositions) within the framework of a vector autoregression. In this Section, we consider two alternative approaches for isolating exogenous variation in macro uncertainty and study its causal impact on capital accumulation.

IA.2.2.1 Structural Uncertainty Shock Identification via Event Constraints Our first identification scheme uses “inequality constraints” on structural shocks, following [Ludvigson, Ma, and Ng \(2021\)](#). Specifically, as in [Ludvigson, Ma, and Ng \(2021\)](#), we apply two sets of restrictions to identify structural uncertainty shocks. The first is a set of event constraints. It requires the identified uncertainty shocks to exhibit certain signs and magnitudes during key historical periods. The broad idea is that a well-supported interpretation of certain episodes involves a particular behavior of the structural uncertainty shocks. The second is a set of

external variable constraints. It mandates that the identified uncertainty shocks exhibit a signed correlation with selected variables that are informative about uncertainty shocks but external to the VAR.

Specifically, consider the VAR system outlined in Equation (9), where the non-core variable y equals to the investment rate, ik . The reduced form innovations in this system are given by $\varepsilon_{\Gamma,t+1}^{aux}$, where the mean of the innovations is zero, and the variance-covariance matrix of the innovations is denoted by Σ^{aux} . Assume that these innovations are related to deep structural shocks $\varepsilon_{t+1}^{struct}$ by an invertible matrix B :

$$\varepsilon_{\Gamma,t+1}^{aux} = B\varepsilon_{t+1}^{struct},$$

for $\varepsilon_{t+1}^{struct} \sim (0, I_{N+1})$ and $BB' = \Sigma^{aux}$. The time series of the structural uncertainty shocks is a function of the matrix B and given by $\varepsilon_v^{struct}(B) = e_v' B^{-1} \varepsilon_{\Gamma,t+1}^{aux}$. The impulse-response from structural uncertainty shocks to some variable x in h periods is obtained by $e_x \Phi^{h-1, aux} B e_v$.

One particular example of such matrix B is the unique lower Cholesky decomposition $P = Chol(\Sigma^{aux})$. This is the case that is used in the main text of the paper, and the uncertainty shock identification is based on recursive ordering. In contrast, [Ludvigson, Ma, and Ng \(2021\)](#) propose a different identification strategy, which is independent of the particular ordering of the variables in the VAR, and hinges on event and external variable constraints.

Let $\mathbb{B}$ be the set of all invertible matrices B such that BB' is equal to the variance-covariance matrix of the innovations. Without loss of generality, $\mathbb{B} = \{B = PQ \mid Q \in O_{N+1 \times N+1}, \text{diag}(B) \geq 0, BB' = \Sigma^{aux}\}$ where $O_{N+1 \times N+1}$ is the set of $N + 1 \times N + 1$ orthonormal matrices. Building on the methodology of [Ludvigson, Ma, and Ng \(2021\)](#), we then require that any admissible time series of structural uncertainty shock, $\varepsilon_v^{struct}(B)$, formed from $B \in \mathbb{B}$ satisfies the following constraints:

1. $\varepsilon_{v, \tau_1}^{struct} \geq \bar{k}_0$ at $\tau_1 = 1951$
2. $\varepsilon_{v, \tau_2}^{struct} \geq \bar{k}_0$ at $\tau_2 = 1953$
3. $\varepsilon_{v, \tau_3}^{struct} \geq \bar{k}_0$ at $\tau_3 = 1965$
4. $\varepsilon_{v, \tau_4}^{struct} \geq \bar{k}_1$ at $\tau_4 = 1971$

5. $\varepsilon_{v,\tau_5}^{struct} \geq \bar{k}_1$ at $\tau_5 = 1979$
6. $\varepsilon_{v,\tau_6}^{struct} \geq \bar{k}_1$ at $\tau_6 = 2011$
7. $Corr(\varepsilon_{v,t}^{struct}, R_{m,t}^e) < 0$ for $t \in [1971, 2022]$
8. $Corr(\varepsilon_{v,t}^{struct}, Gold_t) > 0$ for $t \in [1971, 2022]$

We start by explaining constraints 4 to 8, as they are identical to the identification scheme of strict macro uncertainty shocks by [Ludvigson, Ma, and Ng \(2021\)](#).⁵ Constraints 4 to 6 pertain to large exogenous events, causing an elevation in macro uncertainty. In particular, these constraints require that the macro (TFP) uncertainty shocks in years 1971, 1979, and 2011 exceed $\bar{k}_1$ standard deviations above the mean. Macro uncertainty at the onset of 1971 was driven by the likelihood and consequences of a collapse of the Bretton Woods system (“Nixon” exogenous shock). During 1979, as explained in detail by [Ludvigson, Ma, and Ng \(2021\)](#), Paul Volcker exogenously created higher macro uncertainty: he revealed the outcomes of an impromptu FOMC meeting where a decision was made to replace the longstanding federal funds rate instrument approach with a new nonborrowed reserves instrument approach for implementing monetary policy, aimed at curbing the rising inflation (“Volker Experiment” exogenous shock). In 2011, the U.S. House of Representatives departed from long-established precedent by insisting that the U.S. President engage in negotiations on deficit reduction as a condition for raising the debt ceiling (“Debt Ceiling” exogenous shock). To be conservative, and in line with [Ludvigson, Ma, and Ng \(2021\)](#), we set $\bar{k}_1$ to 0, only requiring positive uncertainty shocks in these periods.

Constraints 7 and 8 are referred to as external variable constraints. [Ludvigson, Ma, and Ng \(2021\)](#) argue that both the stock market and the price of safe-haven assets such as gold are informative about macro uncertainty. Constraint 7 imposes the well-established fact that periods of high uncertainty coincide with lower realized market excess returns. Constraint 8 states

⁵[Ludvigson, Ma, and Ng \(2021\)](#) include other event constraints, beyond constraints 4–6 above, but those are used to identify financial uncertainty shocks, as opposed to macro uncertainty shocks. Macro uncertainty broadly corresponds to the TFP uncertainty in our paper. In particular, the 2008 uncertainty shock is associated with financial rather than macro uncertainty.

that the correlation between identified uncertainty shocks and the (hp-filtered) price of gold is positive. Uncertainty-generating events include natural disasters, terrorist attacks, political coups, and revolutions, as discussed in Baker and Bloom (2013). [Ludvigson, Ma, and Ng \(2021\)](#) argue that plausibly exogenous surges in uncertainty, triggered by such events, have generally corresponded to price appreciation of safe-haven assets like gold. Conditions 7 and 8 pertain to the period starting in 1971, when President Nixon announced the unilateral suspension of the gold convertibility of the dollar. They help to further disentangle first- and second- moment TFP shocks by ensuring the divergence of stock and gold prices in response to the latter.

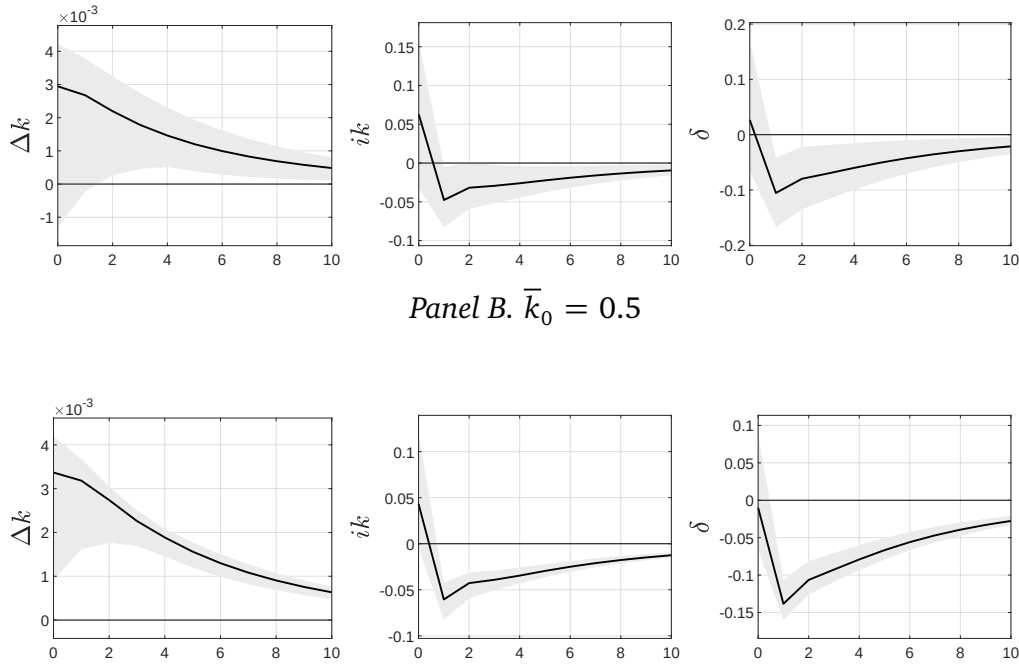
Since our sample period starts in 1948, almost 20 years before the analysis of [Ludvigson, Ma, and Ng \(2021\)](#), we include additional exogenous event restrictions that pertain to the earlier years of the sample. Constraints 1 –3 require that the TFP uncertainty shock found in the periods 1951, 1953, and 1965 exceed $\bar{k}_0$ standard deviations above the mean. The choice of τ_1 and τ_2 corresponds with the Korean War: 1951 was associated with launching of Operation Thunderbolt, whereas 1953 saw the height of the war and was associated with Eisenhower’s election, who campaigned to bring an end to the Korean conflict (“Korean War” and “Eisenhower Election” shocks). The choice of τ_3 corresponds to the Indo-Pakistani war and the onset of the US military involvement in the Vietnam War (“Vietnam War” shock). We deliberately focus on global – and hence plausibly exogenous – shocks to US macro uncertainty. We consider two values for $\bar{k}_0$: First, a conservative value of zero (such that $\bar{k}_0 = \bar{k}_1$). Second, to accommodate the Great Moderation phenomenon, where uncertainty shocks are arguably higher in these earlier years of the sample, we also consider setting $\bar{k}_0$ to 0.5.⁶

To implement the above, we follow [Ludvigson, Ma, and Ng \(2021\)](#). Consider P , the lower triangular Cholesky decomposition of Σ^{aux} . We then rotate it by 3 million random orthogonal matrices Q . Each rotation begins by drawing $N + 1 \times N + 1$ matrix M of $NID(0, 1)$ random variables. Then Q is taken to be the orthonormal matrix in the QR decomposition of M . We

⁶In an untabulated result, we consider a third option of removing constraints 1–3 altogether, such that the remaining constraints (4–8) are identical to [Ludvigson, Ma, and Ng \(2021\)](#). To compensate for the constraints removal – given our longer sample period – we increase the value of $\bar{k}_1$ to a one standard deviation shock. The results are nearly identical to Panel A of Figure [IA.2.3](#).

then set the candidate $B \in \mathbb{B}$ as PQ . Lastly, we check whether the structural uncertainty shocks implied by the candidate B , $\varepsilon_v^{struct}(B)$ satisfy the constraints 1–8 above.

Figure IA.2.3: Uncertainty Shocks Impulse Responses: Event Identification (Ludvigson, Ma, and Ng (2021)) Panel A. $\bar{k}_0 = 0$



The figure shows one-standard-deviation uncertainty shock impulse-responses of log capital growth Δk , investment rate ik , and depreciation rate δ . The identification of structural uncertainty shocks follows an even SVAR identification following Ludvigson, Ma, and Ng (2021), as outlined in Section IA.2.2.1.

IA.2.2.2 Uncertainty Shock Identification via Instruments Denote by $\mathbb{B}_{pass}$ the set of matrices $B \in \mathbb{B}$ that pass the constraints 1 – 8. Using each $B \in \mathbb{B}_{pass}$, we compute the impulse response to capital growth and investment rate, and the implied impulse response to depreciation using Equation (5). Figure IA.2.3 shows the median impulse responses in solid, as well as the confidence intervals implied by the minimum and maximum impulse responses obtained using all the matrices $\mathbb{B}_{pass}$. Consistent with the evidence from Section 1.3.3, we find that structural uncertainty shocks induce a positive and significant impact on capital growth. Under the conservative value of $\bar{k}_1$, this positive effect is significant from year 2 onward (Panel A), whereas under the alternative value, the effect is significant already at time 0 (Panel B). Likewise, in both panels A and B, the structural uncertainty shock leads to a persistent decline

in investment (from time 1 onward) and depreciation rates, with the latter being larger in absolute value.

Our second identification scheme relies on instrumental variables. Consider a linear projection of the future components of capital accumulation on Γ_t , as outlined in Equation (7). In the main text, we identify the impact of uncertainty on h -periods ahead capital growth and investment rate expectations using plain OLS estimates, given by $\beta_{\Delta k}^h e_v$ and $\beta_{ik}^h e_v$, respectively.

In this Section, we consider using an IV approach to instrument macro uncertainty v in projection (7). To this end, we use the proposed instruments of [Baker, Bloom, and Terry \(2024\)](#), and thank the authors for making the instrument data publicly available. [Baker, Bloom, and Terry \(2024\)](#) argue that uncertainty could be instrumented using exogenous disaster shocks, encompassing four categories: (i) natural disasters (e.g., extreme weather, earthquakes, pandemics), (ii) terrorist attacks, (iii) coups, and (iv) political shocks (revolutions in particular). These shocks are arguably exogenous in the short run.

For each disaster category, the authors construct two sets of instruments. The first set is “domestic”: it depends only on disaster information pertaining to a focal country (in our case, the US). The authors use the AccessWorld News Newsbank service to construct “attention” index around each category, creating media weights for these disaster shock instruments. Specifically, they scale each shock by the increase in media coverage fifteen days after the event compared to fifteen days before the event (percentage increase in the number of articles written in the fifteen days after the event compared to the fifteen days before the event).

The second set of instruments is “trade-weighted”. It is motivated by the idea that the economic conditions of a nation may depend not only upon their own shocks but also those of nearby nations. The authors construct alternative disaster instruments, which equal to the sum of the domestic disaster series plus the disaster series of other nations weighted by the bilateral trade to GDP ratio.

Importantly, for the US, the “domestic” instruments that are based on coups and revolutions only take the value of zero (i.e., no such events occurred in the US), allowing us to only use information about natural disasters and terrorist attacks. However, when using “trade-

weighted” instruments, all four categories of instruments are populated for the US economy. The instrument data start in the 1970s and end in 2021. The raw instrument data are quarterly, and we use the average value of each instrument over the last four quarters to obtain annual observations. For the US, the terrorist attack instrument only takes nonzero values in the second half of the sample (related to the 9/11 attack). We therefore consider both the full-sample period as well as a more recent subsample starting in 1985.

Table 3 in the body of the paper reports the responses of expected cumulative capital growth, investment rate, and depreciation to instrumented macro uncertainty, using both the “domestic” and “trade-weighted” instruments, and using both the full- and recent-sample periods. Using the IV approach, the slope coefficients for capital growth are always positive, increase with the predictive horizon, and are statistically significant at horizons 3 to 5 years ahead.

IA.2.3 Uncertainty Shocks and Capital Accumulation:

UK Evidence

We source data on UK TFP and capital accumulation margins from Penn World Table (PWT), compiled by [Feenstra, Inklaar, and Timmer \(2015\)](#). The data are annual and available from 1954 to 2019. The dataset provides information on the growth of the UK TFP (item *ctfp*), the aggregate capital stock (item *rnn*, also available at Fred), and depreciation (item *delta*). Investment can be implicitly computed from the dataset as GDP’s share of gross capital formation (item *csht*) multiplied by GDP (item *cgdp*).

To estimate the volatility of the UK’s TFP, we follow a procedure that is nearly identical to the benchmark methodology, as outlined in Section [IA.1](#), but modified for annual frequency data. Specifically, we first filter the TFP data through an AR(1) process to eliminate persistent variations in productivity and obtain innovations in the TFP. We then filter out the time-varying volatility of these innovations via a Glosten-Jagannathan-Runkle (*GJR*(1, 1)) model.

As in the main analysis for the US, we set Γ_t (core predictors) to include all four state variables in their covariance-stationary form: TFP volatility and the log growth rates of TFP, capital, and one-period lagged depreciation. We note that, unlike BEA data, the depreciation

data provided by PWT only capture the average depreciation rate of the capital stock over each year. As a result, the raw data from PWT on capital growth, investment rate, and depreciation rate do not satisfy the capital accumulation identity. Consequently, we only utilize PWT data on lagged depreciation as a control variable. The effect of uncertainty on the UK depreciation rate is implied from its estimated effects on capital growth and investment rates, using Equations (5) and (6).

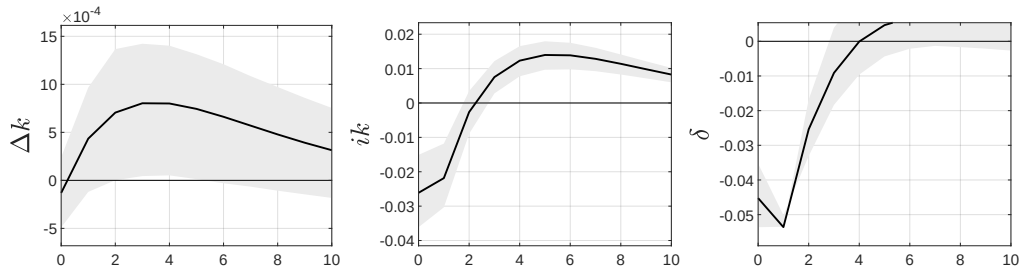
Using UK data, Table IA.2.1 reports the results of projections (7), while Figure IA.2.4 shows uncertainty impulse responses obtained using the same procedure described in Section 1.3.3. Similarly to the US, the slope coefficients on UK macro uncertainty are positive and significant (at the 10% level or higher) for capital growth, but negative and significant for investment and depreciation rates. The impulse response of capital growth is strictly positive and significant at horizons 2 to 6 years ahead. Interestingly, the impulse response of investment rate shows an initial decline followed by an investment overshoot, broadly consistent with Bloom (2009). The implied impulse response of the depreciation rate is strictly negative whenever significant.

Table IA.2.1: Uncertainty and Capital Accumulation: UK Evidence

	UK Macro Uncertainty
$\Delta k_{t \rightarrow t+1}$	0.17 [1.67]
$\Delta k_{t \rightarrow t+3}$	1.00 [5.64]
$\Delta k_{t \rightarrow t+5}$	2.00 [11.66]
ik_{t+1}	-6.43 [-2.21]
ik_{t+3}	-1.43 [-2.35]
ik_{t+5}	-4.96 [-1.73]
δ_{t+1} (implied)	-17.13 [-3.31]
δ_{t+3} (implied)	-15.44 [-2.83]
δ_{t+5} (implied)	-22.57 [-4.85]

The table shows the estimated slope coefficients on UK macroeconomic uncertainty in projections of future cumulative capital growth, Δk , investment rate, ik , and depreciation rate, δ , controlling for the remaining core state variables. All capital accumulation measures pertain to the UK and are obtained from the PWT database. The predictive horizon is 1, 3 and 5 years ahead. For interpretability, the slopes are multiplied by the standard deviation of macro volatility (times 100). Bootstrapped t-statistics are in brackets. Data are annual from 1954 to 2019.

Figure IA.2.4: Uncertainty Shocks Impulse Responses: UK Evidence



The figure shows one-standard-deviation UK uncertainty shock impulse-responses of log capital growth Δk , investment rate ik , and depreciation rate δ (implied). All capital accumulation measures pertain to the UK and are obtained from the PWT database. Data are annual from 1954 to 2019.

IA.2.4 Supplemental Tables and Figures: Empirical Evidence

Table IA.2.2: Correlation Between Alternative Measures

Panel A.

	Uncertainty: Benchmark (GJR)	Uncertainty: SV	Uncertainty: E-GARCH
Uncertainty: Benchmark (GJR)	1		
Uncertainty: SV	0.34	1	
Uncertainty: E-GARCH	0.66	0.38	1

Panel B.

	Prod: Benchmark (TFP)	Prod: Solow(Ex R&D)	Prod: Solow(Equip&Struct)	Prod: Labor Prod
Prod: Benchmark (TFP)	1			
Prod: Solow(Ex R&D)	0.999	1		
Prod: Solow(Equip&Struct)	0.995	0.995	1	
Prod: Labor Prod	0.887	0.886	0.889	1

Panel C.

	Capital: Benchmark (Non-Resid)	Capital: Equip & Struct	Capital: Total Priv	Capital: Compustat
Capital: Benchmark (Non-Resid)	1			
Capital: Equip & Struct	0.971	1		
Capital: Total Priv	0.889	0.867	1	
Capital: Compustat	0.452	0.471	0.309	1

The table shows the correlation between alternative uncertainty measures (Panel A), alternative productivity measures (Panel B), and alternative capital growth measures (Panel C). Data are annual from 1948 to 2022.

Table IA.2.3: Uncertainty and Capital Accumulation: Lagged Capital Growth

	(1)	(2)	(3)	(4)	(5)
	Benchmark	Capital: Equip & Struct	Capital: Total Priv	Capital: Compustat	Volatility: SV
$\Delta k_{t \rightarrow t+1}$	0.24 [3.14]	0.24 [3.76]	0.17 [2.41]	1.78 [4.56]	0.11 [1.08]
$\Delta k_{t \rightarrow t+3}$	0.77 [6.74]	0.85 [10.14]	0.65 [6.15]	2.85 [7.25]	0.64 [5.74]
$\Delta k_{t \rightarrow t+5}$	1.06 [9.56]	1.40 [15.48]	0.94 [9.84]	3.40 [8.43]	1.31 [12.41]
ik_{t+1}	-9.80 [-4.20]	-2.09 [-1.75]	-3.07 [-2.29]	-8.24 [-2.18]	-10.19 [-4.56]
ik_{t+3}	-9.86 [-4.04]	-0.60 [-0.44]	-2.42 [-1.81]	-9.71 [-2.19]	-9.79 [-4.16]
ik_{t+5}	-10.94 [-5.36]	-0.89 [-0.75]	-3.79 [-2.87]	-9.74 [-2.13]	-10.80 [-5.04]
δ_{t+1} (implied)	-16.85 [-4.74]	-7.90 [-5.32]	-9.00 [-3.32]	-15.54 [-5.02]	-14.87 [-4.54]
δ_{t+3} (implied)	-16.85 [-5.17]	-7.25 [-4.78]	-9.25 [-3.05]	-14.78 [-4.98]	-18.28 [-5.77]
δ_{t+5} (implied)	-15.65 [-4.76]	-6.20 [-4.22]	-8.10 [-2.65]	-13.33 [-4.38]	-19.39 [-7.17]
δ_{t+1} (direct)	-11.71 [-4.87]	-6.39 [-5.59]	-6.08 [-3.53]	-	-10.80 [-4.77]
δ_{t+3} (direct)	-11.59 [-5.41]	-6.02 [-5.68]	-6.01 [-3.26]	-	-12.08 [-5.47]
δ_{t+5} (direct)	-11.47 [-5.27]	-5.82 [-5.31]	-6.11 [-3.37]	-	-13.03 [-7.38]
	(6)	(7)	(8)	(9)	(10)
	Volatility: E-GARCH	Productivity: Solow(Ex R&D)	Productivity: Solow(Equip&Struct)	Productivity: Labor Prod	Sample: 1965-
$\Delta k_{t \rightarrow t+1}$	0.19 [1.95]	0.25 [3.20]	0.24 [3.10]	0.07 [0.98]	0.24 [2.05]
$\Delta k_{t \rightarrow t+3}$	0.57 [4.76]	0.79 [6.98]	0.77 [6.47]	0.44 [4.75]	1.13 [9.03]
$\Delta k_{t \rightarrow t+5}$	0.78 [5.76]	1.11 [9.76]	1.04 [9.55]	0.70 [6.83]	1.90 [19.44]
ik_{t+1}	-6.53 [-3.00]	-10.19 [-4.33]	-9.66 [-4.02]	-5.31 [-2.36]	-1.29 [-0.64]
ik_{t+3}	-7.05 [-2.87]	-10.28 [-4.18]	-9.80 [-3.78]	-3.85 [-1.59]	0.18 [0.07]
ik_{t+5}	-7.30 [-2.80]	-11.35 [-5.51]	-11.00 [-5.38]	-5.13 [-2.73]	-0.03 [-0.01]
δ_{t+1} (implied)	-11.90 [2.98]	-17.51 [-4.94]	-16.73 [-4.59]	-7.98 [-2.24]	-5.74 [-1.54]
δ_{t+3} (implied)	-11.85 [-3.43]	-17.53 [-5.49]	-16.71 [-4.87]	-8.51 [-2.48]	-7.81 [-1.89]
δ_{t+5} (implied)	-10.99 [-2.26]	-16.35 [-4.94]	-15.49 [-4.95]	-7.69 [-2.49]	-5.82 [-2.55]
δ_{t+1} (direct)	-8.08 [-3.17]	-12.17 [-5.08]	-11.60 [-4.71]	-5.83 [-2.43]	-3.25 [-1.36]
δ_{t+3} (direct)	-8.17 [-3.55]	-12.06 [-5.66]	-11.52 [-5.06]	-5.47 [-2.24]	-3.82 [-1.42]
δ_{t+5} (direct)	-7.87 [-2.66]	-11.94 [-5.42]	-11.45 [-5.33]	-5.53 [-2.72]	-3.11 [-1.94]

The table shows the estimated slope coefficients on macroeconomic uncertainty in projections of future cumulative capital growth, Δk , investment rate, ik , and depreciation rate, δ , controlling for the remaining core state variables. The capital control is replaced with its lagged value. The predictive horizon is 1, 3 and 5 years ahead. For interpretability, the slopes are scaled by the standard deviation of macro volatility and multiplied by 100. Bootstrapped t-statistics are in brackets. Data are annual from 1948 to 2022.

Table IA.2.4: Uncertainty and Capital Accumulation: HP-Filtered Capital

	(1)	(2)	(3)	(4)	(5)
	Benchmark	Capital: Equip & Struct	Capital: Total Priv	Capital: Compustat	Volatility: SV
$\Delta k_{t \rightarrow t+1}$	0.51 [4.56]	0.47 [4.65]	0.51 [5.51]	2.04 [5.97]	0.33 [2.65]
$\Delta k_{t \rightarrow t+3}$	1.51 [16.80]	1.42 [18.85]	1.55 [20.55]	3.74 [9.70]	1.20 [13.71]
$\Delta k_{t \rightarrow t+5}$	2.30 [22.93]	2.29 [25.36]	2.45 [26.74]	4.78 [12.99]	2.20 [24.51]
ik_{t+1}	-16.79 [-6.10]	-2.91 [-2.62]	-11.05 [-6.12]	-10.75 [-2.86]	-15.97 [-7.09]
ik_{t+3}	-16.49 [-6.44]	-2.22 [-1.90]	-10.65 [-6.01]	-12.01 [-2.64]	-15.99 [-5.69]
ik_{t+5}	-16.61 [-7.23]	-2.27 [-1.99]	-10.84 [-6.93]	-12.29 [-2.61]	-16.36 [-7.32]
δ_{t+1} (implied)	-30.82 [-6.69]	-13.84 [-6.21]	-29.39 [-6.82]	-16.86 [-5.42]	-26.31 [-6.11]
δ_{t+3} (implied)	-29.66 [-8.09]	-12.88 [-7.19]	-28.51 [-8.28]	-15.60 [-5.25]	-29.04 [-7.96]
δ_{t+5} (implied)	-27.40 [-6.83]	-11.61 [-5.72]	-26.38 [-6.85]	-14.48 [-6.30]	-29.45 [-9.62]
δ_{t+1} (direct)	-20.39 [-6.75]	-10.64 [-6.82]	-19.52 [-6.99]	-	-17.96 [-6.35]
δ_{t+3} (direct)	-19.63 [-7.78]	-10.01 [-7.64]	-18.79 [-7.86]	-	-19.09 [-7.79]
δ_{t+5} (direct)	-18.69 [-7.52]	-9.46 [-6.87]	-17.90 [-7.50]	-	-19.48 [-9.14]
	(6)	(7)	(8)	(9)	(10)
	Volatility: E-GARCH	Productivity: Solow(Ex R&D)	Productivity: Solow(Equip&Struct)	Productivity: Labor Prod	Sample: 1965-
$\Delta k_{t \rightarrow t+1}$	0.48 [4.09]	0.52 [4.68]	0.54 [4.85]	0.26 [2.69]	0.51 [5.71]
$\Delta k_{t \rightarrow t+3}$	1.39 [14.77]	1.55 [17.22]	1.60 [18.10]	0.93 [10.04]	1.62 [24.13]
$\Delta k_{t \rightarrow t+5}$	2.13 [18.35]	2.35 [23.38]	2.43 [24.75]	1.45 [14.50]	2.66 [25.97]
ik_{t+1}	-14.04 [-5.17]	-17.27 [-6.31]	-17.34 [-6.43]	-10.07 [-3.54]	-13.47 [-5.39]
ik_{t+3}	-14.18 [-5.70]	-17.01 [-6.78]	-17.02 [-6.66]	-8.39 [-2.98]	-13.27 [-6.18]
ik_{t+5}	-13.53 [-4.37]	-17.10 [-7.55]	-17.22 [-7.50]	-9.29 [-3.30]	-12.25 [-6.57]
δ_{t+1} (implied)	-26.78 [-5.72]	-31.66 [-6.92]	-32.07 [-6.81]	-17.63 [-3.43]	-25.57 [-6.11]
δ_{t+3} (implied)	-25.82 [-6.02]	-30.46 [-8.29]	-30.85 [-8.47]	-16.73 [-3.89]	-26.20 [-8.25]
δ_{t+5} (implied)	-23.73 [-4.48]	-28.23 [-7.16]	-28.51 [-7.11]	-15.36 [-3.47]	-23.92 [-8.40]
δ_{t+1} (direct)	-17.39 [-5.59]	-20.96 [-6.99]	-21.15 [-7.00]	-11.86 [-3.60]	-16.77 [-6.10]
δ_{t+3} (direct)	-16.92 [-6.44]	-20.19 [-8.14]	-20.36 [-7.94]	-10.77 [-3.91]	-17.08 [-7.79]
δ_{t+5} (direct)	-15.76 [-4.65]	-19.25 [-7.93]	-19.43 [-7.80]	-10.45 [-3.57]	-15.81 [-8.49]

The table shows the estimated slope coefficients on macroeconomic uncertainty in projections of future cumulative capital growth, Δk , investment rate, ik , and depreciation rate, δ , controlling for the remaining core state variables. The capital control is replaced with its hp-filtered value. The predictive horizon is 1, 3 and 5 years ahead. For interpretability, the slopes are scaled by the standard deviation of macro volatility and multiplied by 100. Bootstrapped t-statistics are in brackets. Data are annual from 1948 to 2022.

Table IA.2.5: Uncertainty and Capital Accumulation: Expansions vs Recessions

	(1)	(2)
	Expansions	Recessions
$\Delta k_{t \rightarrow t+1}$	0.09 [1.55]	-0.10 [-1.55]
$\Delta k_{t \rightarrow t+3}$	0.49 [4.63]	0.02 [0.17]
$\Delta k_{t \rightarrow t+5}$	0.68 [6.19]	0.36 [3.47]
ik_{t+1}	-9.52 [-3.55]	-11.87 [-4.28]
ik_{t+3}	-8.91 [-3.41]	-9.96 [-3.56]
ik_{t+5}	-9.69 [-5.42]	-9.31 [-4.91]
δ_{t+1} (implied)	-13.76 [-4.15]	-13.07 [-3.69]
δ_{t+3} (implied)	-14.59 [-4.09]	-14.81 [-4.17]
δ_{t+5} (implied)	-13.21 [-4.50]	-13.86 [-4.07]
δ_{t+1} (direct)	-10.22 [-4.20]	-10.62 [-4.01]
δ_{t+3} (direct)	-10.28 [-4.25]	-10.82 [-4.44]
δ_{t+5} (direct)	-9.95 [-5.27]	-10.05 [-4.89]

The table shows the estimated slope coefficients on macroeconomic uncertainty in projections of future cumulative capital growth, Δk , investment rate, ik , and depreciation rate, δ , controlling for the remaining core state variables. Macroeconomic uncertainty is interacted with either an expansion dummy (Column 1) or a recession dummy (Column 2). The predictive horizon is 1, 3 and 5 years ahead. For interpretability, the slopes are scaled by the standard deviation of macro volatility and multiplied by 100. Bootstrapped t-statistics are in brackets. Data are annual from 1948 to 2022.

Table IA.2.6: Uncertainty and Skewness: Capital Accumulation Projections

	(1)	(2)
	β_v^h	β_{skew}^h
$\Delta k_{t \rightarrow t+1}$	0.04 [0.49]	-0.02 [-0.34]
$\Delta k_{t \rightarrow t+3}$	0.56 [5.47]	0.14 [1.70]
$\Delta k_{t \rightarrow t+5}$	1.13 [13.05]	0.92 [9.19]
ik_{t+1}	-5.23 [-2.50]	-1.23 [-0.51]
ik_{t+3}	-0.91 [-0.36]	-0.37 [0.23]
ik_{t+5}	-0.25 [-0.15]	2.50 [1.49]
δ_{t+1} (implied)	-7.25 [-2.79]	-1.14 [-0.39]
δ_{t+3} (implied)	-6.41 [-1.99]	-2.35 [-1.18]
δ_{t+5} (implied)	-4.78 [-2.24]	-3.75 [-1.51]
δ_{t+1} (direct)	-5.09 [-1.96]	-1.09 [-0.38]
δ_{t+3} (direct)	-3.45 [-1.07]	-1.31 [-0.65]
δ_{t+5} (direct)	-2.59 [-1.21]	-1.21 [-0.49]

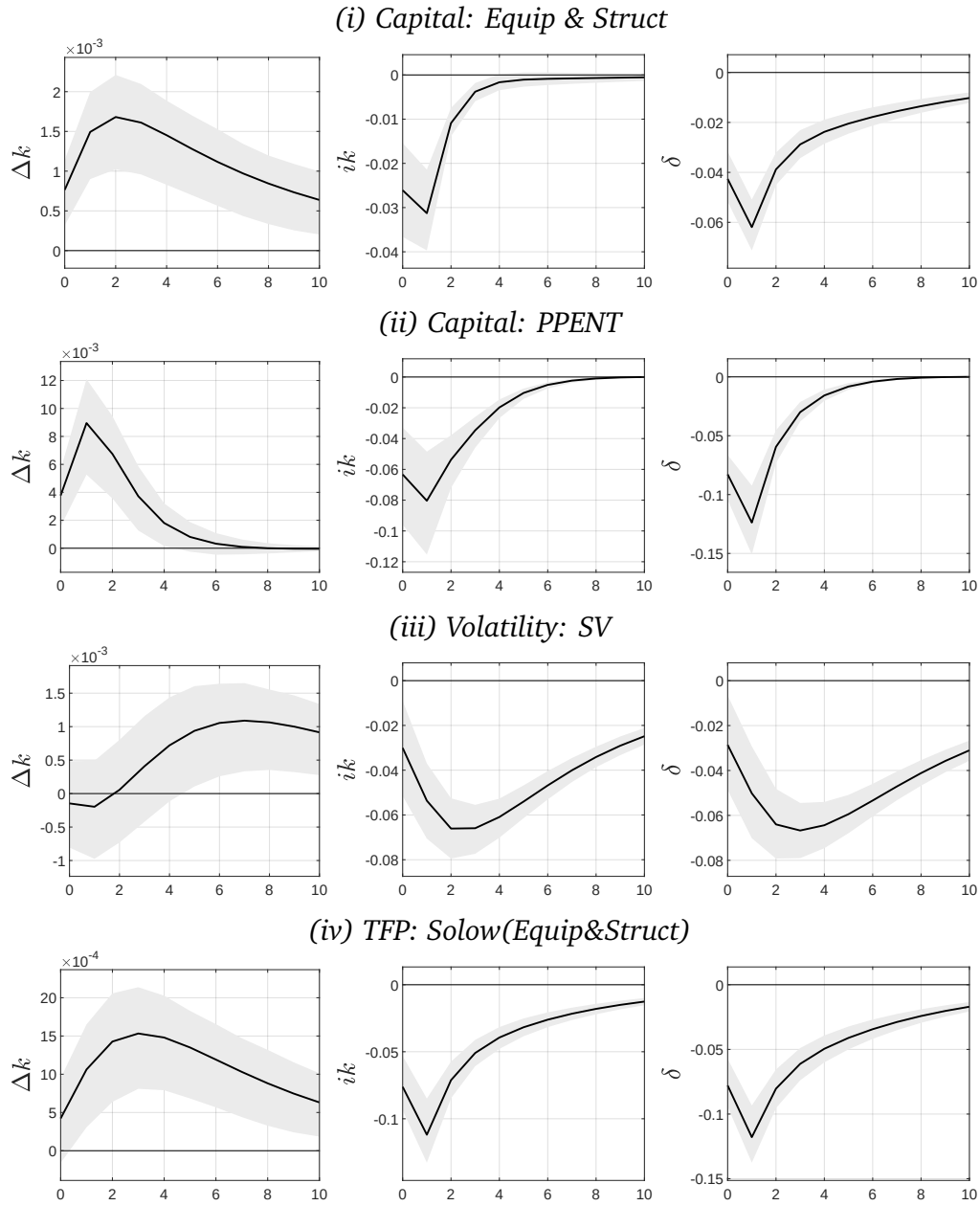
The table shows the estimated slope coefficients on macroeconomic uncertainty and skewness in projections of future cumulative capital growth, Δk , investment rate, ik , and depreciation rate, δ , controlling for the remaining core state variables. The predictive horizon is 1, 3 and 5 years ahead. For interpretability, the uncertainty (skewness) slopes are scaled by the standard deviation of macro volatility (macro skewness) multiplied by a factor of 100. Bootstrapped t-statistics are in brackets. Data are annual from 1963 to 2022.

Table IA.2.7: Persistence in Depreciation and Utilization

Industry	$AC(1)_u$	$AC(1)_\delta$	p-val
Food, beverage, and tobacco	0.55	0.77	< 1%
Textiles and products	0.38	0.90	< 1%
Apparel and leather goods	0.15	0.92	< 1%
Paper	0.24	0.81	< 1%
Printing and related support activities	0.58	0.78	< 1%
Petroleum and coal products	0.72	0.74	< 1%
Chemical	0.49	0.90	< 1%
Plastics and rubber products	0.52	0.86	< 1%
Primary metal	0.49	0.86	< 1%
Fabricated metal product	0.53	0.78	< 1%
Machinery	0.54	0.87	< 1%
Computer and electronic product	0.46	0.93	< 1%
Electrical equipment, appliance, and component	0.49	0.92	< 1%
Motor vehicles and parts	0.57	0.89	< 1%
Aerospace and miscellaneous transportation eq.	0.57	0.91	< 1%
Furniture and related product	0.55	0.89	< 1%
Support activities for mining	0.41	0.50	< 1%
Miscellaneous	0.39	0.85	< 1%

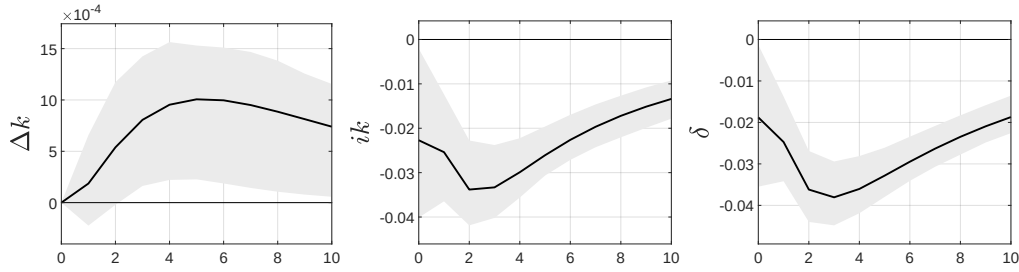
The table shows the estimates of the persistence parameters in depreciation ($AC(1)_\delta$) and utilization rates ($AC(1)_u$), along with the p-value ($p - val$) of a Wald test that the two autocorrelations are identical. Both utilization and depreciation rates are hp-filtered. Data are annual from 1972 to 2022.

Figure IA.2.5: Uncertainty Shock Impulse Responses: Supplemental Evidence



The figure shows one-standard-deviation uncertainty shock impulse responses of log capital growth Δk , investment rate ik , and depreciation rate δ , under alternative configurations, not covered in the body of the paper.

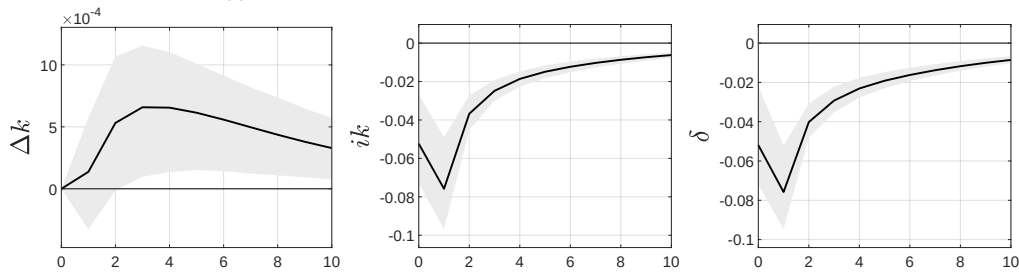
Figure IA.2.6: Uncertainty Shocks Impulse Responses: Skewness in Core



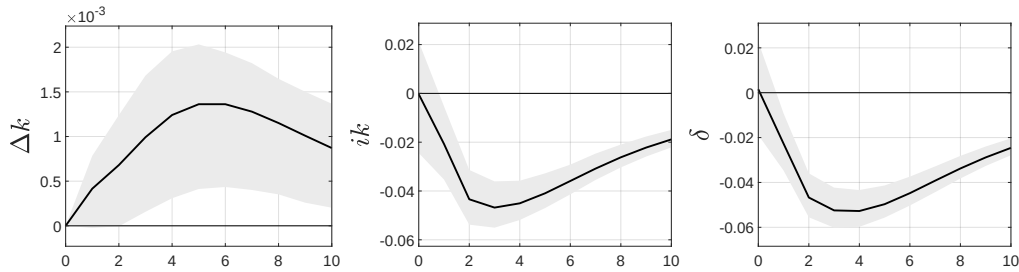
The figure shows one-standard-deviation uncertainty shock impulse-responses of log capital growth Δk , investment rate ik , and depreciation rate δ , when the VAR includes the cross-sectional skewness of firms' TFP as a core variable in the vector Γ , and uncertainty is order last such that uncertainty shocks are orthogonal from skewness. The analysis starts in 1963 due to the availability of firm-level TFP data.

Figure IA.2.7: Uncertainty Shocks Impulse Responses: Frictions in the Core

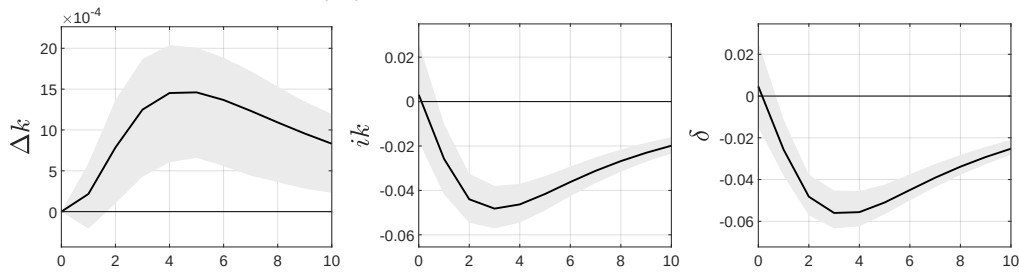
(i) Core variables include excess market return



(ii) Core variables include GZ-index

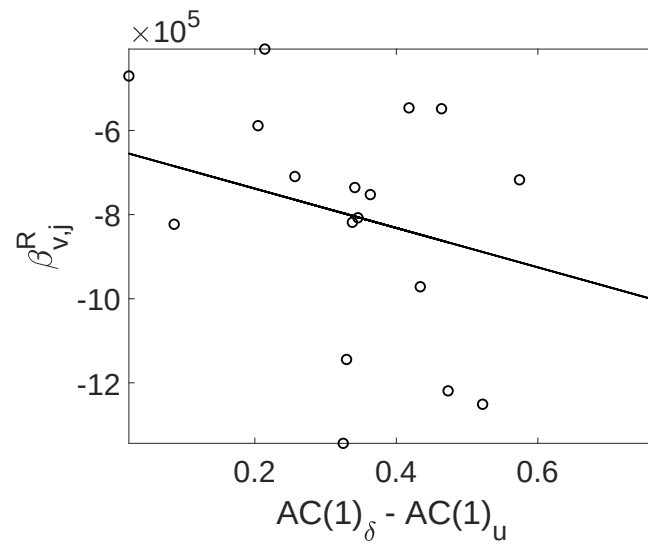


(iii) Core variables include EBP



The figure shows one-standard-deviation uncertainty shock impulse responses of log capital growth Δk , investment rate ik , and depreciation rate δ , when the VAR includes in the core-variables Γ : (i) Excess market return, (ii) GZ-index, (iii) Excess bond premium (EBP). The analysis for (ii) and (iii) starts in 1973 due to data availability. In all cases, uncertainty is ordered last such that uncertainty shocks are orthogonal to financial frictions.

Figure IA.2.8: Uncertainty Risk Exposures and Excess Persistence in Depreciation



The figure shows a scatter plot of industry j return beta to uncertainty risks, $\beta_{v,j}^R$, against industry j difference in the persistence of depreciation and utilization, $[AC(1)_\delta - AC(1)_u]_j$. Solid line indicates best linear fit.

IA.3 Additional Model Results

IA.3.1 Equilibrium Definition and Optimality Conditions

Equilibrium. In the benchmark model, an equilibrium consists of the wage W_t , pricing kernel $M_{t,t+1}$, firm valuation V_t , and allocations for investment, capital, labor, utilization, depreciation, consumption, and equity holding $\{I_t, K_{t+1}, L_t, u_t, \delta_t, C_t, \Theta_t\}_{t=0}^{\infty}$ such that: (i) Given W_t and $M_{t,t+1}$, capital, utilization, and labor allocations maximize program (16), (ii) Given W_t and V_t , consumption, labor and firm holding fraction maximize (17) subject to (18), (iii) goods market clears: $C_t + I_t = Y_t$, $\forall t$, labor market clears: $L_t = 1$, $\forall t$, and financial market clears: $\Theta_t = 1$, $\forall t$.

Optimality conditions. Labor choice is static and satisfies the standard first-order condition:

$$(1 - \alpha) \frac{Y_t}{L_t} = W_t. \quad (\text{IA.1})$$

The Euler condition for pricing any return R_j satisfies:

$$1 = \mathbb{E}_t [M_{t,t+1} R_{j,t+1}]. \quad (\text{IA.2})$$

In particular, the investment choice, I_t , is determined using the Euler equation:

$$1 = \mathbb{E}_t [M_{t,t+1} R_{t+1}^I], \quad (\text{IA.3})$$

where R_{t+1}^I denotes the returns to investment given by:

$$R_{t+1}^I = \frac{MPK_{t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1} (1 - \delta_{t+1} + \phi(\frac{I_{t+1}}{K_{t+1}}))}{q_t}. \quad (\text{IA.4})$$

$MPK_{t+1} = \alpha Y_{t+1} / K_{t+1}$ is the marginal product of capital at time $t + 1$, and Tobin's marginal q is:

$$q_t = \phi' \left(\frac{I_t}{K_t} \right)^{-1}. \quad (\text{IA.5})$$

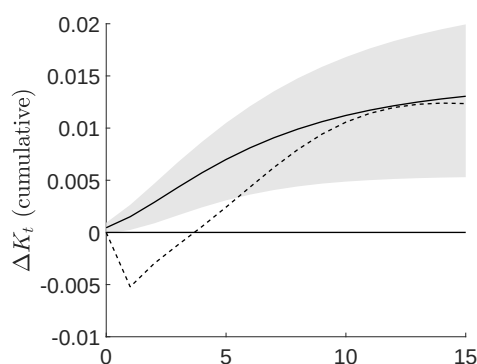
Since q_t measures the present value of an extra unit of installed capital, Equation (IA.3) shows the trade-off between the marginal cost and discounted marginal benefit of buying capital. Note that MPK_{t+1} depends positively on u_{t+1} .

IA.3.2 Are Capital Growth Dynamics Realistic?

The capital impulse response depicted in Figure 2 is monthly and pertains to detrended capital. Consequently, it cannot be directly compared to the empirical responses, which are

annual and are expressed in growth rates. To help assess whether the model generates realistic dynamics of capital accumulation, we feed the model with an uncertainty shock that generates an empirically equivalent one-year-ahead decline in depreciation and track the model-implied response of the capital. We sample the model-implied observations every four quarters so that the frequency of the observations is annual. Lastly, we cumulate the uncertainty impact on capital growth in both the model and the data.

Figure IA.3.9: Uncertainty shocks Capital Growth: model vs data



The figure shows model-implied uncertainty impulse response (dashed) and empirical impulse response (solid) to the cumulative growth rate of the capital stock ΔK

Figure IA.3.9 shows the results. The data response, shown in a solid line, is identical to panel (v) of Figure 1. The dashed line depicts the model-implied response, and shaded area indicates empirical confidence intervals. Recall that the structural estimation of the model only utilizes unconditional moments of the data, and does not target the dynamic effects. Nonetheless, the model provides a reasonable fit to the data at longer horizons. The impulse response is within the empirical confidence interval after five years, and the long-term dynamics converge almost perfectly to those observed in the data.⁷

⁷The model predicts that capital falls in the short run as the persistent depreciation and utilization effects on capital have not caught up with immediate drop in investment. Note that in half of the empirical impulse responses in Figure 1, the confidence intervals include a possibility of negative capital growth at short horizons.

IA.3.3 Capital and Investment Divergence:

A Falsification Test

The evidence in Section 1.3.2 shows that the slope coefficient on macro uncertainty is positive for future capital growth but negative for future investment rates. Put differently, states of higher macro uncertainty are linked to divergent expectations. Whereas higher uncertainty is associated with higher expectations about future capital growth, it is also associated with lower expectations about future investment rates.

A general question about the projection results is whether this divergence in expectations arises from exogenous variation in uncertainty or because of endogenous responses to first-moment shocks. In particular, while projections (7) control for the growth rate in TFP, large first-moment shocks alone could generate a fluctuation in realized volatility. Coupled with persistence in TFP, which also creates persistence in capital accumulation measures, it leads to the following question: Can first-moment shocks alone lead to a simultaneous increase in volatility and a divergence of future capital growth and investment rate expectations?

One way to tackle this question is empirical: using event-based identification of structural uncertainty shocks (Section [IA.2.2.1](#)) or using an instrumented variable approach (Section [IA.2.2.2](#)). A complementary approach is theoretical, using a falsification test, which we describe below.

We perform the following experiment using the model from Section 2:

1. Shut down stochastic volatility in the model by setting σ_w to zero.
2. Simulate finite paths and aggregate them to an annual frequency having the same lengths as the data.
3. For each path, obtain innovations of the annual TFP growth rate from an AR(1) model, and feed these innovations into a *GJR* – *GARCH* model to filter TFP volatility. This is similar to the construction of the uncertainty measure in the empirical analysis. In this case,

fluctuations in “uncertainty” are only driven by random large and persistent first-moment shocks — which are identified as uncertainty fluctuations in finite samples.

4. Project future cumulative capital growth rates and future investment rates on the model-implied Γ control variables (i.e., TFP volatility, and the annual growth in TFP, capital and depreciation). This is equivalent to projection (7) in the empirical section.
5. Combining all finite sample simulation results, compile the model-implied “population” slope coefficients on TFP volatility.
6. Perform a “falsification” test: Check whether the slope coefficients for future capital growth and investment rates have the opposite signs, despite having no exogenous shocks to stochastic volatility.

We then repeat steps 2–6 above under the benchmark model, where exogenous shocks to uncertainty are permitted.

The results are reported in Table [IA.3.8](#). First, under the benchmark model, and across all predictive horizons, the population slope coefficients for future capital growth are positive, while they are negative for future investment rates. While the magnitude of the model-implied coefficients are not identical to the empirical estimates, they share a similar order of magnitude (see Table 2). For example, the slope coefficient for 3-year ahead capital growth is 0.95 and 1.54 in the model and in the data, respectively. Likewise, the slope coefficient for 3-year ahead investment rate is -38.37 and -37.48 in the model and the data, respectively. Importantly, the model-implied slopes share the same signs as the empirical counterparts, indicating that with exogenous uncertainty shocks, the model can reproduce the divergent pattern of capital and investment expectations in states of elevated uncertainty.

Second, when the dynamic environment is only characterized by endogenous responses to stochastic first-moment fluctuations ($\sigma_w = 0$), two patterns emerge. On one hand, the slope coefficients for future investment rates are mostly negative — this is the case for the 3- and 5-year ahead projections. Although these slopes are smaller than the data, the negative signs echo the general concern in the uncertainty literature that at least some part of the

future decline in investment in states of higher uncertainty is potentially endogenous, and not attributed to exogenous uncertainty fluctuations. On the other hand, for all predictive horizons, the slope coefficients for future capital growth rates share the same sign as the slope coefficients for future investment rates. In other words, a decline in TFP (or contemporaneous investment) could plausibly lead to higher measured volatility and to lower future investment rates in finite samples. At the same time, this would result in future capital growth moving in the same direction as investment. From the perspective of our structural model, the observed uncertainty-related patterns cannot be obtained in the absence of exogenous second-moment shocks.

Table IA.3.8: Model-Implied Divergence of Capital and Investment: Exogenous Uncertainty versus Only First-Moment Shocks

Horizon(k)	Exogenous Uncertainty (Benchmark)			No Exogenous Uncertainty ($\sigma_w = 0$)		
	$\Delta k_{t \rightarrow t+k}$	ik_{t+k}	Opposite Sign?	$\Delta k_{t \rightarrow t+k}$	ik_{t+k}	Opposite Sign?
1	0.05	-87.16	Yes	0.00	0.04	No
3	0.95	-38.37	Yes	-0.37	-2.74	No
5	4.03	-5.98	Yes	-1.22	-23.69	No

The table shows the population values for the slope coefficients on macroeconomic uncertainty in model-implied projections of future cumulative capital growth, Δk , and investment rate, ik , controlling for the remaining core state variables. The predictive horizon is 1, 3 and 5 years ahead. For comparability across models, the model-implied filtered TFP-volatility is re-scaled in each simulation to feature the same standard deviation as the benchmark.

IA.3.4 Comparison to the New-Keynesian model

Basu and Bundick (2017) and Fernández-Villaverde et al. (2015) show that a New-Keynesian model featuring time-varying markups can induce a comovement between consumption and investment in response to uncertainty shocks. In this section, we extend our baseline model, featuring flexible utilization and persistent depreciation, to accommodate countercyclical markups.

We show that under our baseline parameter values and permanent productivity shocks, time-varying markups alone do not cause investment to decline following an uncertainty shock. However, when we incorporate extra persistence in depreciation, the impulse responses to consumption, investment, and output are all negative. We also show that under transitory

productivity shocks, time-varying markups can make investment drop in response to uncertainty. When persistent depreciation is added in this case, the impulse responses to consumption and output are significantly magnified.

Augmented Model. We present an augmented model that accommodates monopolistic competition and nominal rigidity.

The household side of the economy is identical to that described in Section 2. Unlike the perfect competition model, the economy is populated by a mass of differentiated intermediate goods producers, indexed by $m \in [0, 1]$. The output of an intermediate good producer at time t of variety m is denoted by $y_t(m)$ and its price is $p_t(m)$.

Aggregator. An aggregator converts the intermediate goods into a final composite layer good, Y_t , using a CES production function:

$$Y_t = \left[\int_0^1 y_t(m)^{\frac{\theta-1}{\theta}} dm \right]^{\frac{\theta}{\theta-1}}. \quad (\text{IA.6})$$

For any finite θ , the intermediate goods are not perfect substitutes, and producers possess some amount of monopolistic power.

The aggregator faces perfect competition in the product market. It solves:

$$\max_{\{y_t(m)\}} P_t Y_t - \int_0^1 p_t(m) y_t(m) dm \quad \text{s.t equation (IA.6)}.$$

It follows that the price index is given by $P_t = \left[\int_0^1 p_t(m)^{1-\theta} dm \right]^{\frac{1}{1-\theta}}$, and the demand schedule for each intermediate good producer of variety m is $\left[\frac{p_t(m)}{P_t} \right]^{-\theta} Y_t$.

The aggregator supplies final goods used for either consumption or investment by the intermediate good producers.

Intermediate good producers. The intermediate good producer of variety m faces the same capital accumulation and production technology as described in Section 2.1. It owns its capital stock, $K_t(m)$, which depreciates at rate $\delta_t(m)$, and it hires labor from the household. It has an additional degree of freedom and can optimally select its nominal output price.

The real dividend of an intermediate good producer of variety m , $d_t(m)/P_t$, is given by:

$$d_t(m)/P_t = \left[\frac{p_t(m)}{P_t} \right]^{1-\theta} Y_t - I_t - W_t/P_t \cdot L_t(m) - \phi_P/2 \left(\frac{p_t(m)}{\Pi p_{t-1}(m)} - 1 \right)^2 Y_t, \quad (\text{IA.7})$$

where ϕ_p captures the degree of nominal rigidity as in [Rotemberg \(1982\)](#). Each intermediate good producer chooses its output price, optimal hiring, utilization rate, and investment, to maximize its market value, taking as given wages W_t , the price index P_t , and the stochastic discount factor of the household $M_{t,t+1}$. Specifically, each firm maximizes:

$$V_t(m) = \max_{\{L_s(m), p_s(m), K_{s+1}(m), u_s(m)\}} E_t \sum_{s=t}^{\infty} M_{t,s} (d_s(m)/P_s) \quad (\text{IA.8})$$

s.t.

$$\begin{aligned} \left[\frac{p_t(m)}{P_t} \right]^{-\theta} Y_t &\leq A_t^{1-\alpha} (u_t(m) K_t(m))^\alpha L_t(m)^{1-\alpha} \\ K_{t+1}(m) &= (1 - \delta_t(m)) K_t(m) + \phi \left(\frac{I_t(m)}{K_t(m)} \right) K_t(m) \\ \delta_t(m) &= (1 - \rho_\delta) \delta_0 + \rho_\delta \delta_{t-1}(m) + \varepsilon_\delta(u_t(m)), \end{aligned}$$

where $\varepsilon_\delta(u_t(m))$ is given by Equation (13).

Monetary Policy. A monetary authority sets the nominal interest rate r_t according to the following rule:

$$r_t = r_{ss} + \rho_\pi (\pi_t - \pi_{ss}) + \rho_y (\Delta y_t - \Delta y_{ss}),$$

where $r_t = \log(R_t)$, $\pi_t = \log(\Pi_t) = \log(P_t/P_{t-1})$, and $\Delta y = \log(Y_t/Y_{t-1})$. π_{ss} and Δy_{ss} are the steady-state log inflation and log output growth, respectively.

Aggregate productivity. We consider two separate dynamics for aggregate productivity. The first case corresponds to permanent productivity shocks to A_{t+1} :

$$\Delta a_{t+1} = \mu + e^{\sigma_{a,t}} \tilde{\sigma}_a \varepsilon_{a,t+1}.$$

The second case corresponds to transitory productivity shocks to A_{t+1} :

$$A_{t+1} = \mu^{t+1} \exp(e^{\sigma_{a,t}} \tilde{\sigma}_a \varepsilon_{a,t+1}).$$

In both cases, the stochastic volatility $\sigma_{a,t}$ follows the dynamics described in equation (21).

Market clearing and equilibrium. The market clearing conditions of the labor markets, and the goods markets are modified as follows:

$$\int_0^1 L_t(m) dm = 1, \quad \int_0^1 I_t(m) dm + C_t = Y_t.$$

Equilibrium consists of prices, labor, utilization, and capital allocations such that (i) taking prices and wages as given, the household's allocation maximizes (17), and firms' allocations

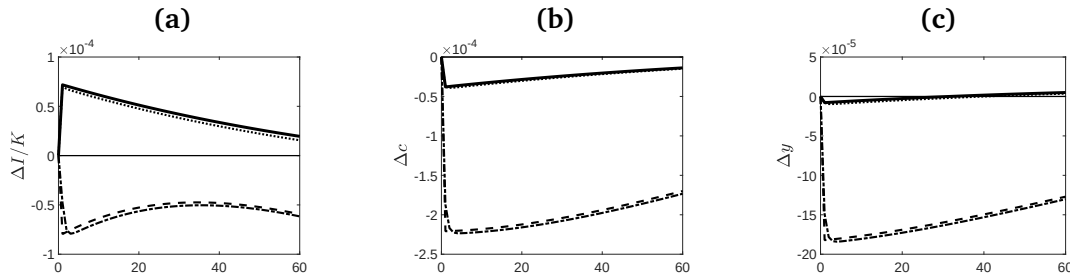
solve (IA.8); (ii) all markets clear; (iii) we are interested in a symmetric equilibrium in which $K_t(m) = K_t$, $u_t(m) = u_t$, $L_t(m) = L_t$, and $p_t(m) = p_t$, for all $m \in [0, 1]$.

Calibration. All model parameters are identical to those specified in Section 2.5, with the following additions and modifications: (i) following Basu and Bundick (2017) we set $\rho_\pi = 1.5$, $\rho_y = 0.2$, $\theta = 6$, $\phi_p = 100$; (ii) $\tilde{\sigma}_a = \sqrt{\sigma_a^2/(1-\rho_a^2)}$ (the values of σ_a and ρ_a are identical to Table 4).

Impulse responses. Figures IA.3.10 and IA.3.11 show the model-implied uncertainty impulse responses under the case of permanent and transitory productivity shocks, respectively. We obtain the following results:

- (1) With either permanent or transitory productivity shocks, our mechanism of flexible utilization and persistent depreciation alone (i.e., $\rho_\delta = 0.99$ but $\phi_p = 0$) is sufficient to induce comovement between consumption, investment, and output following an uncertainty shock. All real quantities decrease in the presence of fixed but positive markups.
- (2) Under permanent productivity shocks, time-varying markups alone (i.e., $\rho_\delta = 0$ but $\phi_p = 100$) reduce investment compared to the case of fixed markups ($\phi_p = 0$). However, quantitatively, investment still rises in response to an uncertainty shock, despite utilizing the same values for θ and ϕ_p as in Basu and Bundick (2017).
- (3) Under permanent productivity shocks, when we combine our channel ($\rho_\delta = 0.99$) with the time-varying markup channel ($\phi_p = 100$) the drop in investment, consumption, and output is only slightly amplified compared to the case of persistent depreciation and fixed markups. About 95% of the drop in investment is due to the persistent depreciation, while 5% is due to the rise in markups.
- (4) Under transitory productivity shocks, the time-varying markup channel alone (i.e., $\rho_\delta = 0$ but $\phi_p = 100$) is sufficiently strong as to drop consumption, investment, and output, following an uncertainty shock. This is consistent with the results of Basu and Bundick

Figure IA.3.10: Uncertainty shocks IRF: New-Keynesian model with permanent productivity shocks



The figure shows model-implied uncertainty impulse responses to (a) investment rate $\Delta I/K$, (b) consumption c , and (c) output y . We consider calibrations with no persistent depreciation and no price stickiness ($\rho_\delta = 0$, $\phi_p = 0$, solid line); persistent depreciation but no price stickiness ($\rho_\delta = 0.99$, $\phi_p = 0$, dashed line); no persistent depreciation but with price stickiness ($\rho_\delta = 0$, $\phi_p = 100$, dotted line); persistent depreciation and price stickiness ($\rho_\delta = 0.99$, $\phi_p = 100$, dotted-dashed line). In all specifications, utilization is flexible and the average markups are identical. The productivity shocks have a permanent effect on productivity level ($\Delta a_{t+1} = \mu + e^{\sigma_{a,t}} \sigma_a \varepsilon_{a,t+1}$). All growth impulse responses are cumulative.

(2017), which study the time-varying volatility of mean-reverting shocks. When time-varying markups are combined with persistent depreciation (i.e., $\rho_\delta = 0.99$ and $\phi_p = 100$), about 60% of the drop in consumption and output is due to time-varying markups, and 40% of the drop is due to the depreciation dynamics. Thus, our economic channel provides a significant amplification to the economic channel of Basu and Bundick (2017).

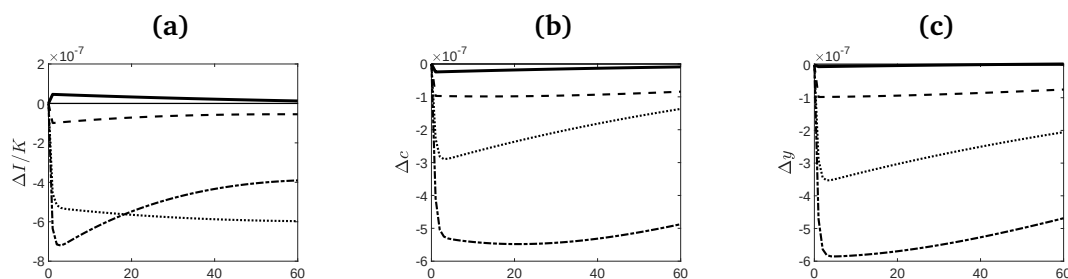
IA.3.5 Supplemental Tables and Figures: Model

Table IA.3.9: Return Predictability using Realized Volatility: Model and Data

Benchmark								
Horizon	Model		$\rho_\delta = 0$		$\zeta \rightarrow \infty$		Data	
	Coeff.	R^2	Coeff.	R^2	Coeff.	R^2	Coeff.	R^2
1y	12.11	0.01	7.88	0.01	7.74	0.01	15.51 [2.45]	0.02
2y	16.34	0.03	10.52	0.01	10.45	0.01	20.90 [4.59]	0.04
3y	18.87	0.04	12.24	0.01	12.18	0.01	24.44 [4.96]	0.06
4y	20.62	0.04	13.41	0.02	13.37	0.02	24.37 [4.13]	0.06
5y	22.16	0.05	14.43	0.02	14.33	0.02	27.60 [6.15]	0.08

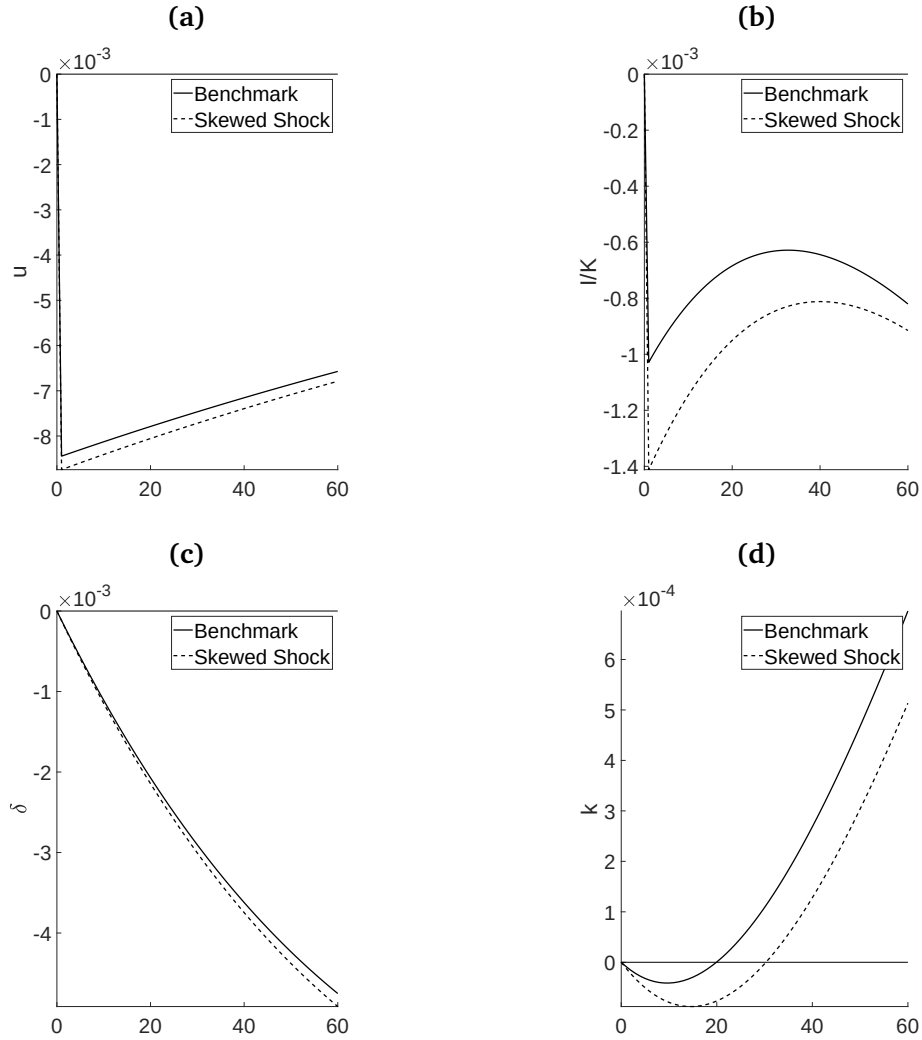
The table shows the slope coefficients on volatility and the R^2 s in the following projection: $R_{t+1 \rightarrow t+k}^e = \text{const} + \beta_{rv} RV_t + \text{error}$, where $R_{t \rightarrow t+h}^e$ is the future market excess return up to horizon h and RV_t is the current 12-month rolling window realized volatility of market returns. The left column shows the model results under the benchmark parameters. The second column shows the results when $\rho_\delta = 0$ (no extra persistence in depreciation). The next column shows the results when $\zeta \rightarrow \infty$ (fixed utilization). The last column represents the data. The model results are based on a population sample of 100,000 periods. Newey-West t-statistics are in brackets. Data are from 1948 to 2022.

Figure IA.3.11: Uncertainty shocks IRF: New-Keynesian model with transitory productivity shocks



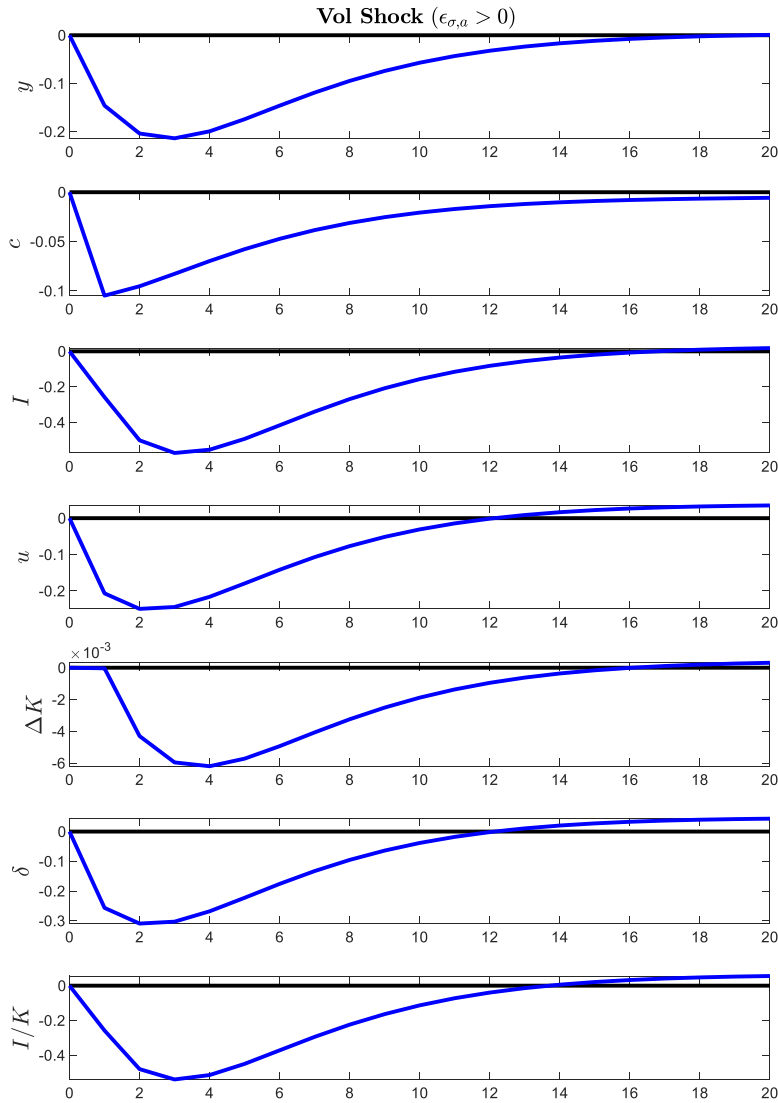
The figure shows model-implied uncertainty impulse responses to (a) investment rate $\Delta I/K$, (b) consumption c , and (c) output y . We consider calibrations with no persistent depreciation and no price stickiness ($\rho_\delta = 0$, $\phi_p = 0$, solid line); persistent depreciation but no price stickiness ($\rho_\delta = 0.99$, $\phi_p = 0$, dashed line); no persistent depreciation but with price stickiness ($\rho_\delta = 0$, $\phi_p = 100$, dotted line); persistent depreciation and price stickiness ($\rho_\delta = 0.99$, $\phi_p = 100$, dotted-dashed line). In all specifications, utilization is flexible and the average markups are identical. The productivity shocks have a permanent effect on productivity level ($A_{t+1} = \mu^{t+1} \exp(e^{\sigma_{a,t}} \sigma_a \varepsilon_{a,t+1})$). All growth impulse responses are cumulative.

Figure IA.3.12: Model-implied IR to uncertainty shocks: Benchmark vs Skewed Shock



The figure shows the impulse responses of (a) utilization rate u , (b) investment rate I/K , (c) depreciation rate δ , (d) detrended capital stock K . The solid line shows the results using the benchmark model parameters. The dotted line shows the results when the shock is $\varepsilon_a^{skew} \equiv \varepsilon_a - 0.017\varepsilon^2$, matching the skewness of utilization-adjusted TFP to the data. The horizontal axis represents months. The vertical axis represents deviations from the steady-state values.

Figure IA.3.13: Basu and Bundick (2017): Depreciation, and capital growth



The figure shows the replicated impulse responses to uncertainty shocks under the benchmark calibration of Basu and Bundick (2017) model. The vertical axis represents percent deviation from the steady state value. The first three IRFs for output y , consumption c , and investment I , are reported by authors in Figure 3. The other IRFs for utilization u , capital growth ΔK , depreciation rate δ , and investment rate I/K are based on the replication code, extended to accommodate these IRFs.

IA.4 Utilization and Deprecation: Supplemental Evidence

IA.4.1 Empirical Relationship

Our key finding that uncertainty shocks are associated with an increase in the stock of capital does not hinge on establishing an empirical nexus between depreciation and utilization. Nonetheless, our economic explanation assumes a positive and persistent link between capacity utilization and future depreciation rates, which we can directly assess in the data.

We examine this relation by running a regression of future cumulative depreciation growth rate at horizon H on the current depreciation growth and the current utilization rate u_t :

$$\frac{1}{H}\Delta\delta_{t-1 \rightarrow t+H-1} = \text{const} + \beta_{\delta,H}\Delta\delta_{t-1} + \beta_{u,H}u_t + \text{error},$$

where $\Delta\delta_{t-1 \rightarrow t+H-1} = \sum_{h=1}^H \Delta\delta_{t-1+h}$. Both the dependent and independent variables are standardized for ease of interpretation.

Panel (a) of Table [IA.4.10](#) shows the estimates of the slope coefficients together with the Newey-West t-statistics. In the first row of the table, the control $\Delta\delta_{t-1}$ is omitted, and the reported $\beta_{u,1}$ is equal to the correlation between $\Delta\delta_t$ and u_t . The estimated correlation is 0.50, and it is economically and statistically significant. The predictive coefficient $\beta_{u,H}$ remains positive and significant at longer horizons so that the predictive effect of utilization on future depreciation growth is persistent over time. Notably, the lagged depreciation rate remains a positive and significant predictor of its future growth, controlling for the current utilization rate. Indeed, β_{δ} is positive and significant at least at a 10% confidence level across all the horizons. This suggests that the current utilization rate alone does not fully capture persistent fluctuations in the depreciation rate, which is consistent with our modeling choice. When $H = 1$, the value of β_{δ} is 0.6 at an annual frequency, or $0.6^{1/12} = 0.96$ at the monthly frequency. This is very close to the model-implied estimate of ρ_{δ} .⁸

⁸The former regression uses the growth of depreciation given that it is highly persistent in the data. For completeness, we repeat the analysis using the depreciation level as a dependent and an independent variable. We obtain similar results: utilization rates predict future depreciation levels with p-value < 5%.

Table IA.4.10: Depreciation and utilization

H	β_u	β_δ
0	0.50 [4.78]	
1	0.44 [5.57]	0.60 [5.91]
2	0.48 [4.59]	0.41 [2.61]
3	0.36 [2.48]	0.32 [1.69]

Panel (a) of the table shows the results of the regression: $\frac{1}{H} \Delta \delta_{t-1 \rightarrow t+H-1} = \text{const} + \beta_{\delta, H} \Delta \delta_{t-1} + \beta_{u, H} u_t + \text{error}$. δ is the depreciation rate, and u is the utilization rate. The control $\Delta \delta_{t-1}$ is omitted in the first row. Newey-West t-statistics are in brackets. Data are annual from 1968 to 2022. All variables are standardized.

IA.4.2 Measurement

We clarify how the measurement practices by Compustat and the BEA could give rise to the evidence from the former subsection.

Compustat-based measures. Admittedly, capital and depreciation from Compustat reflect accounting practices and are thus imperfect proxies for economic measures. At the same time, accounting codes do allow firms to adjust depreciation rates based on their capital utilization, in line with Equation (14). Broadly, Accounting Standard Codification 360 permits firms to select “*methods of apportioning depreciation cost...[to be] based on activity*”, which promotes a continuous connection of utilization and depreciation. Accounting Standard Codification 250 of US GAAP specifically states that “*changes in accounting estimates result from new information...items for which estimate [changes are] necessary are uncollectible receivables, service lives and salvage values.*” Similar provisions are available in the IFRS standard. These accounting practices directly allow for a depreciation-utilization nexus.

BEA-based measures. The BEA aims to provide an economic measure of capital depreciation. It defines depreciation as “*the decline in value due to wear and tear, obsolescence, accidental damage, and aging,*” and conceptualizes it akin to the consumption of fixed capital. Further, economic depreciation is forward-looking, rather than historical: “*As an asset ages, its price changes because it declines in efficiency, or yields fewer productive services, in the current period and in all future periods. Depreciation reflects the present value of all such current and future changes in productive services*” (see [Fraumeni \(1997\)](#)).

The BEA uses geometric depreciation patterns for most asset types because they closely align with the actual profile of price declines in the data. To better capture the economic losses in asset value, the BEA employs two techniques: (a) whenever possible, it uses separate depreciation rates across different types of assets; (b) it incorporates available empirical evidence on asset prices in resale markets.⁹

Consequently, the BEA depreciation level could, in principle, be indirectly affected by utilization through its impact on secondary market prices. Moreover, as we explain in the following subsection, the heterogeneity in the depreciation rate across types of capital and across sectors alone could lead the *aggregate* depreciation rate to persistently correlate with lagged utilization, even if *capital-specific* depreciation rates do not vary over the business cycle.

IA.4.3 Microfoundations

Our model posits a *dynamic* link between utilization and the *aggregate* depreciation rate. In this section, we discuss how such a link can be further microfounded in a model featuring one (or more) of the following factors:

- (i) **Composition of capital goods.** In an economy with different types of capital, the effective depreciation rate takes an average across multiple varieties of capital according to their weight in the firm portfolios. Business-cycle fluctuations can induce simultaneous changes in the utilization and the composition of capital, leading to a lead-lag dependence between utilization and depreciation captured by Equation (14).

As a concrete example, according to BEA measurements, equipment has a lower economic depreciation than R&D capital. Assume that when uncertainty increases, firms stall research on new blueprints and generate revenue from existing product lines. By construction, the capital utilization drops, as installed R&D-related equipment (e.g., software and artistic) is idle. Simultaneously, firms stop new orders pertaining to research expenditures, so R&D goods are gradually depleted. This causes the weight of R&D capital to decrease and that of physical capital to increase. The changes are long-lasting because each weight

⁹Practical challenges for measuring economic depreciation are further discussed in [Hulten and Wykoff \(1980\)](#); [Fraumeni \(1997\)](#); [Giandrea et al. \(2021\)](#).

depends on a stock variable. Consequently, a current drop in utilization would lead to a persistent decline in future depreciation.

A similar argument can be made for other types of capital with different depreciation levels, such as equipment relative to inventory. Uncertainty shocks may halt production of goods using installed machines and push firms to make sales the existing inventory. Inventory goods depreciate faster, so that the firm total depreciation rate declines as inventory is being depleted. We present a simple model with **two types** of capital in Section IA.5 to fix these ideas.

To illustrate the plausibility of these economic channels in the data, we utilize the data from the BEA and Fernald (2014) and construct the relative weight of physical equipment to R&D goods, $w_t^{relative} = \frac{w_t^{equipment}}{w_t^{equipment} + w_t^{R\&D}}$. We use two definitions of equipment: machines only, and equipment plus structures. We estimate the impulse response functions of uncertainty shocks for this relative weight from the auxiliary VAR in Equation (9). The results are shown in Figure IA.4.14. Uncertainty shocks indeed predict a very persistent increase in the relative weight of equipment lasting beyond ten years.^{10 11}

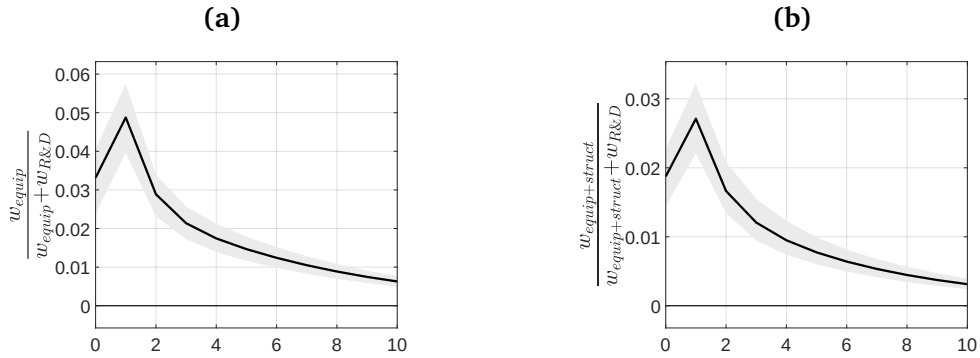
- (ii) **Capital reallocation.** According to the BEA, the depreciation rate of general equipment depends on the sector in which it is used.¹² Moreover, the reallocation of equipment is not instantaneous. There is a time-to-build delay associated with shifting capital from one sector to another. These margins can generate a dynamic lead-lag relationship between the depreciation and utilization rates. On the one hand, reallocation of capital (following

¹⁰The estimates from Figure IA.4.14 suggest that after an uncertainty shock ($H = 1$), the relative weight of equipment rises by about 5%. If the depreciation of equipment is approximately 8% on average per annum, while the depreciation of R&D and software is about 40% on average, a back-of-an-envelope computation implies that the percentage fall in the aggregate depreciation rate relative to its steady state value is almost 20% ($= \left| \frac{0.08 \cdot 1.05 \cdot w_{ss}^{relative} + 0.4 \cdot 0.95 \cdot (1 - w_{ss}^{relative})}{0.08 w_{ss}^{relative} + 0.4(1 - w_{ss}^{relative})} - 1 \right|$). This decline is, in fact, more sizable than the drop in investment rate following higher uncertainty, as implied by Figure 1.

¹¹Figure IA.4.15, we also show that uncertainty shocks decrease the ratio of intellectual property stock over total non-residential stock.

¹²To substantiate this point, Figure IA.4.16 shows a snapshot of the BEA annual depreciation estimates in the year 2013 for general equipment. The exhibit shows that the general equipment used in the investment sector (e.g., machine production) has a lower depreciation than in the consumption sector (e.g., paper production). Even if the depreciation rate within each sector is fixed over time, reallocation of capital across sectors can induce business-cycle fluctuations for the aggregate rate.

Figure IA.4.14: Uncertainty shocks and Relative Weight of Equipment



The figure shows one-standard-deviation uncertainty shock impulse responses to the relative weights of physical equipment, based on the Benchmark configuration. In panel (a), equipment encompasses machines only, whereas in panel (b), equipment also includes structures.

increased uncertainty) lowers utilization contemporaneously: the equipment cannot be fully productive while being reallocated. On the other hand, reallocation simultaneously induces a persistent and potentially permanent effect on the depreciation rate of the reallocated equipment due to sectoral fixed effects.

Notably, our model is cashless and does not feature a storage vehicle. However, the logic above can be extended to instances in which capital is reallocated from a productive sector to a safe storage facility (with little or no depreciation, such as cash). Such reallocation results in the same interaction between utilization and future depreciation.

(iii) **Vintage effects.** To capture economic depreciation, the BEA continuously updates equipment service life to incorporate technological progress and better maintenance. Changes in service lives can induce a persistent effect on future depreciation rates under the realistic assumption of complementary capital vintages.¹³

¹³Indeed, suppose that Equation (10) was augmented to accommodate separate capital vintages and complementary between the vintages:

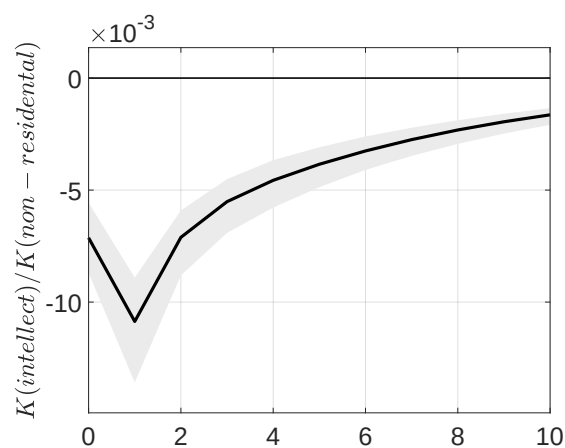
$$Y_t = [(a_{t-\tau,t}K_{t-\tau})^\chi + (a_{t-\tau+1,t}K_{t-\tau+1})^\chi \dots + (u_t K_t)^\chi]^\frac{\alpha}{\chi} L_t^{1-\alpha},$$

where χ captures the elasticity of substitution across vintages. If a vintage of capital acquired τ periods ago is over-utilized, it would reduce its capital-embodied productivity for all future periods ($(a'_{t-\tau,t-\tau+k}(u_t) < 0)$, $k > 1$). This implies that all capital vintages bought afterward would also have to be over-utilized in order to produce any unit of output due to the complementary with the time- t vintage, whose productivity is below the steady state. This persistence in the over-utilization of future vintages induces a persistent effect on depreciation dynamics.

Since we focus on aggregate moments, our model features a single sector and a single type of capital. As an aggregate representation of the economy, Equation (14) captures in a reduced form the rich dependence between utilization and aggregate depreciation originating from (i)–(iii) above.

IA.4.4 Utilization and Deprecation: Additional Evidence

Figure IA.4.15: Uncertainty shocks and Stock of Intellectual Stock to Non-residential Stock



The figure shows one standard deviation uncertainty shock impulse-responses, under the Benchmark configuration, to the ratio of intellectual property capital stock over total non-residential capital stock.

Figure IA.4.16: General equipment depreciation rate in 2013

BEA Rates of Depreciation, Service Lives, Declining-Balance Rates, and Hulten-Wyckoff Categories--Table Continues

Type of Asset	Rate of depreciation	Service life	Declining balance rate	Hulten-Wyckoff category/1/
General industrial, including materials handling equipment:				
Nonmanufacturing industries	0.1072	16	1.715	A
Durable manufacturing:				
Wood products	0.1429	12	1.715	A
Nonmetallic mineral products	0.0903	19	1.715	A
Primary metals	0.0635	27	1.715	A
Fabricated metal products	0.0715	24	1.715	A
Machinery	0.0686	25	1.715	A
Computer and electronic products	0.1225	14	1.715	A
Electronic equipment, appliances, and components	0.1225	14	1.715	A
Motor vehicles, bodies and trailers, and parts	0.1225	14	1.715	A
Other transportation equipment	0.1009	17	1.715	A
Furniture and related products	0.1225	14	1.715	A
Miscellaneous manufacturing:				
Medical equipment and supplies	0.1225	14	1.715	A
Other	0.1009	17	1.715	A
Nondurable manufacturing:				
Food and beverage and tobacco products:				
Food	0.0858	20	1.715	A
Beverage and tobacco product	0.0817	21	1.715	A
Textile and textile product mills	0.1072	16	1.715	A
Apparel and leather and allied products	0.1143	15	1.715	A
Paper products	0.1072	16	1.715	A
Printing and related support activities	0.1143	15	1.715	A
Petroleum and coal products	0.0780	22	1.715	A
Chemical	0.1072	16	1.715	A

The figure shows the example of depreciation rates of general equipment across economic sectors of usage. BEA table for 2013 year.

IA.5 Auxiliary Two-Capital Model

In this subsection, we lay out a simple two-capital model that endogenizes the persistent link between utilization and aggregate depreciation through multiple types of capital goods. A viable option of microfounding equation (14) involves a setup with two types of capital, K_1 and K_2 , where $\delta_1 > \delta_2$, and where an uncertainty shock leads not only to a reduction in utilization, but also to a contemporaneous increase in K_2 at the expense of K_1 . We present a stylized model of this sort, fixing the two types of capital to be machines and inventory, for illustrative purposes.

Assume that a firm has a stock of installed equipment capacity K_t , and a stock of final inventory goods N_t . Inventory depreciates at a rate δ_N , and equipment at a rate of δ_K , where $\delta_N > \delta_K$. The firm's overall depreciation rate, from an accounting standpoint, is given by:

$$\delta_t = \underbrace{\frac{K_t}{K_t + N_t}}_{w_{K,t}} \delta_K + \frac{N_t}{K_t + N_t} \delta_N. \quad (\text{IA.9})$$

All sales are made using the final inventory stock. The firm can replenish its final inventory using production and its installed capacity. The production function is given by $Y_t = A_t(u_t K_t)^\alpha$, where A_t is productivity, and u_t is the utilization rate of equipment.

The firm bears two types of costs. First, there are concave installation costs of new inventory (such as storage and transpiration) $\Lambda(Y_t)$, where $\Lambda' > 0$ and $\Lambda'' < 0$. Second, there is a production cost, proportional to the level of output, at the amount of $f \cdot Y_t$. To illustrate the key point, the setup abstracts from investment, such that all future revenues come from assets-in-place, and all dividends are discounted at a constant factor β .

To summarize, the firm problem can be written recursively as follows:

$$\begin{aligned} V_t(K_t, N_t, A_t) &= \max_{u_t, N_{t+1}} N_t - f \cdot Y_t + \beta \mathbb{E}[V_{t+1}(K_{t+1}, N_{t+1}, A_{t+1})] \\ &s.t. \\ N_{t+1} &= (1 - \delta_N)N_t + \Lambda(Y_t) \\ K_{t+1} &= (1 - \delta_K)K_t \\ Y_t &= A_t(u_t K_t)^\alpha \end{aligned}$$

It is straightforward to show the following optimality condition:

$$\frac{1}{\Lambda'(A_t(u_t K_t)^\alpha)} = \beta \mathbb{E} \left[\frac{1}{f} + \frac{1 - \delta_N}{\Lambda'(A_{t+1}(u_{t+1} (1 - \delta_K) K_t)^\alpha)} \right]. \quad (\text{IA.10})$$

To analyze the impact of a mean-preserving spread over future productivity on utilization, note that since $\Lambda'' < 0$, greater uncertainty over A_{t+1} , all else equal, decreases the right-hand side of equation (IA.10) – intuitively, the expression $1/\Lambda'(\cdot)$ is concave. To equate (lower) the left-hand side, u_t must decline as A_t and K_t are predetermined, and $\Lambda'' < 0$.

It follows that after an uncertainty shock, utilization drops, production declines, and the next-period stock of inventory declines relative to the steady state. From equation (IA.9) we

know that the relative weight of equipment, $w_{K,t}$, increases and therefore δ_t falls. However, in the absence of further shocks, N_{t+k} remains below its steady-state level, and so δ_{t+k} declines as well. In all, an increase in uncertainty is accompanied by $\downarrow u_t$, and $\downarrow \delta_{t+k}$, in line with the intuition of Equation (14).

References

- Alvarez, F., and U. J. Jermann. 2005. Using asset prices to measure the persistence of the marginal utility of wealth. *Econometrica* 73:1977–2016.
- Baker, S. R., N. Bloom, and S. J. Terry. 2024. Using disasters to estimate the impact of uncertainty. *Review of Economic Studies* 91:720–47.
- Basu, S., and B. Bundick. 2017. Uncertainty shocks in a model of effective demand. *Econometrica* 85:937–58.
- Basu, S., J. G. Fernald, and M. S. Kimball. 2006. Are technology improvements contractionary? *American Economic Review* 96:1418–48.
- Bloom, N. 2009. The impact of uncertainty shocks. *Econometrica* 77:623–85.
- Chan, J. C., and C. Y. Hsiao. 2014. Estimation of stochastic volatility models with heavy tails and serial dependence. *Bayesian inference in the social sciences* 155–76.
- Feenstra, R. C., R. Inklaar, and M. P. Timmer. 2015. The next generation of the Penn World Table. *American Economic Review* 105:3150–82.
- Fernald, J. 2014. A quarterly, utilization-adjusted series on total factor productivity. Working Paper, Federal Reserve Bank of San Francisco.
- Fernández-Villaverde, J., P. Guerrón-Quintana, K. Kuester, and J. Rubio-Ramírez. 2015. Fiscal volatility shocks and economic activity. *American Economic Review* 105:3352–84.
- Fraumeni, B. 1997. The measurement of depreciation in the us national income and product accounts. *Survey of Current Business-United States Department of Commerce* 77:7–23.
- Giandrea, M. D., R. J. Kornfeld, P. B. Meyer, S. G. Powers, et al. 2021. Alternative capital asset depreciation rates for us capital and multifactor productivity measures. *Working Paper* .
- Hulten, C. R., and F. C. Wykoff. 1980. *The measurement of economic depreciation*. Citeseer.
- Ludvigson, S. C., S. Ma, and S. Ng. 2021. Uncertainty and business cycles: Exogenous impulse or endogenous response? *American Economic Journal: Macroeconomics* 13:369–410.
- Rotemberg, J. J. 1982. Sticky prices in the united states. *Journal of Political Economy* 90:1187–211.
- Stock, J. H., and M. W. Watson. 2002. Has the business cycle changed and why? *NBER macroeconomics annual* 17:159–218.